E priedas. Trijų laisvės laipsnių modelio analitinės išraiškos

Visos sistemos kinetinė energija Ke=Ke$_1$+Ke$_2$+Ke$_3$:

$$Ke = \frac{m_1\left(\left(-l_{oc1}\sin(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t)\right)^2 + l_{oc1}^2\cos(\theta_{1z}(t))\sigma_3\right) + \left(m_3\left(\sigma_2 + l_2\cos(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_{2z}(t) + l_{bc3}\cos(\theta_{3z}(t))\frac{\partial}{\partial t}\theta_{3z}(t)\right)^2\right) + \left(\sigma_1 + l_2\sin(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_{2z}(t) + l_{bc3}\sin(\theta_{3z}(t))\frac{\partial}{\partial t}\theta_{3z}(t)\right)^2 m_2\left(\left(\sigma_2 + l_{ac2}\cos(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_{2z}(t)\right)^2 + \left(\sigma_1 + l_{ac2}\sin(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_{2z}(t)\right)^2\right)}{2} + \frac{\left(J_1\sigma_3 + J_2\left(\frac{\partial}{\partial t}\theta_{2z}(t)\right)^2 J_3\left(\frac{\partial}{\partial t}\theta_{3z}(t)\right)^2\right)}{2}$$

Kur atskiri kinetinės energijos Ke nariai:

$$\sigma_1 = l_1\sin(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t)$$

$$\sigma_2 = l_1\cos(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t)$$

$$\sigma_3 = \left(\frac{\partial}{\partial t}\theta_{1z}(t)\right)^2$$

Sistemos Lagranžianas L kaip skirtumas tarp kinetinės ir potencinės energijų:

$$L = \frac{\begin{gathered} m_1\left(\left(-l_{oc1}\sin(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t)\right)^2 + l_{oc1}^2\cos(\theta_{1z}(t))^2\sigma_4\right) + \\ m_3\left(\begin{gathered}\left(\sigma_2 + l_2\cos(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_{2z}(t) + l_{bc3}\cos(\theta_{3z}(t))\frac{\partial}{\partial t}\theta_{3z}(t)\right)^2 + \\ \left(\sigma_1 + l_2\sin(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_{2z}(t) + l_{bc3}\sin(\theta_{3z}(t))\frac{\partial}{\partial t}\theta_{3z}(t)\right)^2\end{gathered}\right) + \\ m_2\left(\begin{gathered}\left(\sigma_2 + l_{ac2}\cos(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_{2z}(t)\right)^2 + \\ \left(\sigma_1 + l_{ac2}\sin(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_{2z}(t)\right)^2\end{gathered}\right)\end{gathered}}{2} +$$

$$\frac{J_1\sigma_4}{2} + \frac{J_2\left(\frac{\partial}{\partial t}\theta_{2z}(t)\right)^2}{2} + \frac{J_3\left(\frac{\partial}{\partial t}\theta_{3z}(t)\right)^2}{2} - gm_2(\sigma_3 + l_{ac2}\cos(\theta_{2z}(t))) -$$

$$gm_3(\sigma_3 + l_2\cos(\theta_{2z}(t)) + l_{bc3}\cos(\theta_{3z}(t))) - gl_{oc1}m_1\cos(\theta_{1z}(t))$$

Kur atskiri Lagranžiano L nariai:

$$\sigma_1 = l_1\sin(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t)$$

$$\sigma_2 = l_1\cos(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t)$$

$$\sigma_3 = l_1\cos(\theta_{1z}(t))$$

$$\sigma_4 = \left(\frac{\partial}{\partial t}\theta_{1z}(t)\right)^2$$

Sudaromas matematinis modelis – 3-jų lygčių nehomogeninių antros eilės paprastųjų diferencialinių lygčių sistema.
Pirmajai apibendrintajai koordinatei θ_{1z}:

$$
\begin{aligned}
& m_2 \begin{pmatrix}
2l_1\sin(\theta_{1z}(t))(\sigma_{10}-\sigma_3+l_{ac2}\cos(\theta_{2z}(t))\sigma_2+\sigma_9+l_{ac2}\sin(\theta_{2z}(t))\sigma_1) \\
-2l_1\cos(\theta_{1z}(t))(\sigma_8+l_{ac2}\sin(\theta_{2z}(t))\sigma_2-\sigma_{11}-l_{ac2}\cos(\theta_{2z}(t))\sigma_1) \\
-l_1\sin(\theta_{1z}(t))\left(\sigma_5+l_{ac2}\cos(\theta_{2z}(t))\frac{\partial}{\partial_t}\theta_2(t)\right)\frac{\partial}{\partial_t}\theta_{1z}(t)2 \\
+l_1\cos(\theta_{1z}(t))\left(\sigma_4+l_{ac2}\sin(\theta_{2z}(t))\frac{\partial}{\partial_t}\theta_2(t)\right)\frac{\partial}{\partial_t}\theta_{1z}(t)2
\end{pmatrix} / 2 \\
& - m_3 \begin{pmatrix}
2l_1\cos(\theta_{1z}(t))\begin{pmatrix}\sigma_8+l_2\sin(\theta_{2z}(t))\sigma_2+l_{bc3}\sin(\theta_{3z}(t))\sigma_6-\sigma_{11} \\ -l_2\cos(\theta_{2z}(t))\sigma_1-l_{bc3}\cos(\theta_{3z}(t))\sigma_7\end{pmatrix} \\
-2l_1\sin(\theta_{1z}(t))\begin{pmatrix}\sigma_{10}-\sigma_3+l_2\cos(\theta_{2z}(t))\sigma_2+l_{bc3}\cos(\theta_{3z}(t))\sigma_6+\sigma_9+l_2\sin(\theta_{2z}(t))\sigma_1 \\ +l_{bc3}\sin(\theta_{3z}(t))\sigma_7\end{pmatrix} \\
+l_1\sin(\theta_{1z}(t))\left(\sigma_5+l_2\cos(\theta_{2z}(t))\frac{\partial}{\partial_t}\theta_2(t)+l_{bc3}\cos(\theta_{3z}(t))\frac{\partial}{\partial_t}\theta_{3z}(t)\right)\frac{\partial}{\partial_t}\theta_{1z}(t)2 \\
-l_1\cos(\theta_{1z}(t))\left(\sigma_4+l_2\sin(\theta_{2z}(t))\frac{\partial}{\partial_t}\theta_{2z}(t)+l_{bc3}\sin(\theta_{3z}(t))\frac{\partial}{\partial_t}\theta_{3z}(t)\right)\frac{\partial}{\partial_t}\theta_{1z}(t)2
\end{pmatrix} / 2 \\
& + m_1 \begin{pmatrix}
2l_{ocl}\sin(\theta_{1z}(t))(l_{ocl}\cos(\theta_{1z}(t))\sigma_{12}-\sigma_3+l_{ocl}\sin(\theta_{1z}(t))\sigma_{13}) \\
+2l_{ocl}\cos(\theta_{1z}(t))^2\sigma_{13}-4l_{ocl}\sin(\theta_{1z}(t))\cos(\theta_{1z}(t))\sigma_{12} \\
-2l_{ocl}\cos(\theta_{1z}(t))\left(-l_{ocl}\sin(\theta_{1z}(t))\frac{\partial}{\partial_t}\theta_{1z}(t)\right)\frac{\partial}{\partial_t}\theta_{1z}(t)
\end{pmatrix} / 2 \\
& + J_1\sigma_{13}-gl_1m_2\sin(\theta_{1z}(t))-gl_1m_3\sin(\theta_{1z}(t))-gl_{ocl}m_1\sin(\theta_{1z}(t))=\tau_{1z}-\tau_{2z}
\end{aligned}
$$

Kur:

$$\sigma_1=\frac{\partial^2}{\partial t^2}\theta_{2z}(t),\ \sigma_2=\left(\frac{\partial}{\partial t}\theta_{2z}(t)\right)^2,\ \sigma_3=\frac{\partial}{\partial t}\theta_{3z}(t),\sigma_4=l_1\sin(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t)$$

$$\sigma_5=l_1\cos(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t),\ \sigma_6=\left(\frac{\partial}{\partial t}\theta_{3z}(t)\right)^2,\sigma_7=\frac{\partial^2}{\partial t^2}\theta_{3z}(t),\sigma_8=l_1\sin(\theta_{1z}(t))\sigma_{12},\sigma_9=l_1\sin(\theta_{1z}(t))\sigma_{13}$$

$$\sigma_{10}=l_1\cos(\theta_{1z}(t))\sigma_{12},\sigma_{11}=l_1\cos(\theta_{1z}(t))\sigma_{13},\sigma_{12}=\left(\frac{\partial}{\partial t}\theta_{1z}(t)\right)^2,\sigma_{13}=\frac{\partial^2}{\partial t^2}\theta_{1z}(t)$$

Antrajai apibendrintajai koordinatei, θ_{2z}

$$
\begin{aligned}
& m_2\left(\begin{array}{l}
2l_{ac2}\sin(\theta_{2z}(t))(\sigma_9-\sigma_6+l_{ac2}\cos(\theta_{2z}(t))\sigma_2+\sigma_{10}+l_{ac2}\sin(\theta_{2z}(t))\sigma_1)\\
-2l_{ac2}\cos(\theta_{2z}(t))(\sigma_8+l_{ac2}\sin(\theta_{2z}(t))\sigma_2-\sigma_{11}-l_{ac2}\cos(\theta_{2z}(t))\sigma_1)\\
-l_{ac2}\sin(\theta_{2z}(t))\left(\sigma_4+l_{ac2}\cos(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_2(t)\right)\frac{\partial}{\partial t}\theta_{2z}(t)2\\
+l_{ac2}\cos(\theta_{2z}(t))\left(\sigma_3+l_{ac2}\sin(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_2(t)\right)\frac{\partial}{\partial t}\theta_{2z}(t)2
\end{array}\right)/2\\
& -m_3\left(\begin{array}{l}
2l_2\cos(\theta_{2z}(t))\left(\begin{array}{l}\sigma_8+l_2\sin(\theta_{2z}(t))\sigma_2+l_{bc3}\sin(\theta_{3z}(t))\sigma_5-\sigma_{11}\\-l_2\cos(\theta_{2z}(t))\sigma_1-l_{bc3}\cos(\theta_{3z}(t))\sigma_7\end{array}\right)\\
-2l_2\sin(\theta_{2z}(t))\left(\begin{array}{l}\sigma_9-\sigma_6+l_2\cos(\theta_{2z}(t))\sigma_2+l_{bc3}\cos(\theta_{3z}(t))\sigma_5+\sigma_{10}+l_2\sin(\theta_{2z}(t))\sigma_1\\+l_{bc3}\sin(\theta_{3z}(t))\sigma_7\end{array}\right)\\
+l_2\sin(\theta_{2z}(t))\left(\sigma_4+l_2\cos(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_2(t)+l_{bc3}\cos(\theta_{3z}(t))\frac{\partial}{\partial t}\theta_3(t)\right)\frac{\partial}{\partial t}\theta_{2z}(t)2\\
-l_2\cos(\theta_{2z}(t))\left(\sigma_3+l_2\sin(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_2(t)+l_{bc3}\sin(\theta_{3z}(t))\frac{\partial}{\partial t}\theta_{3z}(t)\right)\frac{\partial}{\partial t}\theta_{2z}(t)^2
\end{array}\right)/2\\
& +J_2\sigma_1-gl_2m_3\sin(\theta_{2z}(t))-gl_{ac2}m_2\sin(\theta_{2z}(t))=\tau_{2z}-\tau_{3z}
\end{aligned}
$$

Kur:

$$\sigma_1=\frac{\partial^2}{\partial t^2}\theta_{2z}(t),\ \sigma_2=\left(\frac{\partial}{\partial t}\theta_{2z}(t)\right)^2,\ \sigma_3=l_1\sin(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t),\ \sigma_4=l_1\cos(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t)$$

$$\sigma_5=\left(\frac{\partial}{\partial t}\theta_{3z}(t)\right)^2,\ \sigma_6=0,\ \sigma_7=\frac{\partial^2}{\partial t^2}\theta_{3z}(t),\ \sigma_8=l_1\sin(\theta_{1z}(t))\sigma_{12},\ \sigma_9=l_1\cos(\theta_{1z}(t))\sigma_{12}$$

$$\sigma_{10}=l_1\sin(\theta_{1z}(t))\sigma_{13},\ \sigma_{11}=l_1\cos(\theta_{1z}(t))\sigma_{13},\ \sigma_{12}=\left(\frac{\partial}{\partial t}\theta_{1z}(t)\right)^2,\ \sigma_{13}=\frac{\partial^2}{\partial t^2}\theta_{1z}(t)$$

Trečiajai apibendrintajai koordinatei, θ_{3z}

$$
\begin{aligned}
& J_3\sigma_1-\left(m_3\left(\begin{array}{l}
2l_{bc3}\cos(\theta_{3z}(t))\left(\begin{array}{l}l_1\sin(\theta_{1z}(t))\sigma_6+l_2\sin(\theta_{2z}(t))\sigma_5+l_{bc3}\sin(\theta_{3z}(t))\sigma_4-l_1\cos(\theta_{1z}(t))\sigma_3\\-l_2\cos(\theta_{2z}(t))\sigma_2-l_{bc3}\cos(\theta_{3z}(t))\sigma_1\end{array}\right)\\
-2l_{bc3}\sin(\theta_{3z}(t))\left(\begin{array}{l}l_1\cos(\theta_{1z}(t))\sigma_6+l_2\cos(\theta_{2z}(t))\sigma_5+l_{bc3}\cos(\theta_{3z}(t))\sigma_4\\+l_1\sin(\theta_{1z}(t))\sigma_3+l_2\sin(\theta_{2z}(t))\sigma_2+l_{bc3}\sin(\theta_{3z}(t))\sigma_1\end{array}\right)\\
+l_{bc3}\sin(\theta_{3z}(t))\left(\begin{array}{l}l_1\cos(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_{1z}(t)+l_2\cos(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_{2z}(t)\\+l_{bc3}\cos(\theta_{3z}(t))\frac{\partial}{\partial t}\theta_{3z}(t)\end{array}\right)\frac{\partial}{\partial t}\theta_{3z}(t)2-l_{bc3}\cos(\theta_{3z}(t))\\
\left(l_1\sin(\theta_{1z}(t))\frac{\partial}{\partial t}\theta_1(t)+l_2\sin(\theta_{2z}(t))\frac{\partial}{\partial t}\theta_2(t)+l_{bc3}\sin(\theta_{3z}(t))\frac{\partial}{\partial t}\theta_3(t)\right)\frac{\partial}{\partial t}\theta_{3z}(t)2
\end{array}\right)\right)/2\\
& -gl_{bc3}m_3\sin(\theta_{3z}(t))=\tau_{3z}
\end{aligned}
$$

Kur:

$$\sigma_1=\frac{\partial^2}{\partial t^2}\theta_{3z}(t),\ \sigma_2=\frac{\partial^2}{\partial t^2}\theta_{2z}(t),\ \sigma_3=\frac{\partial^2}{\partial t^2}\theta_{1z}(t),\ \sigma_4=\left(\frac{\partial}{\partial t}\theta_{3z}(t)\right)^2,\ \sigma_5=\left(\frac{\partial}{\partial t}\theta_{2z}(t)\right)^2,\ \sigma_6=\left(\frac{\partial}{\partial t}\theta_{1z}(t)\right)^2$$