Annex A. The Process of TOPSIS Application

This method is called TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) (Roszkowska, 2011).

Suppose the values of each indicator are continually increasing or decreasing. It is then possible to determine the "ideal" solution that consists of the best indicator values and the "negatively ideal" solution that consists of the worst indicator values. To apply the proximity point approach, it is necessary to construct a decision matrix X.

Step 1: In order to perform a TOPSIS analysis and calculate the weights of the criteria. It is important to perform a methodology or analysis of the application of expert reviews and opinions on the given matrix's weights.

Step 2: Construct the decision matrix and determine the weight of the criteria.

$$X = (x_{IJ}), \quad \text{(A1)}$$

$$W = [w_1, w_2, \dots, w_n], \quad \text{(A2)}$$

where:

X – decision matrix;

W – weight vector, $x_{ij} \in \Re$, and $w_1+w_2+\ldots+w_n=1$.

Criteria of the functions can be benefit functions (more is better) or cost functions (less is better).

Step 3. Calculate the normalized decision matrix. The study of normalization's influence consists of two steps: Analysis of the normalization rules I for a sequence of even pseudo-random numbers. Normalizations were performed for this sequence of numbers, and the scattering characteristics of the normalized sequences were monitored; II – By changing the normalization rules in the TOPSIS method, the results obtained are subjected to statistical analysis.

The second step is the calculation of the normalization of matrix transforms different criteria dimensions into non-dimensional. This allows creating a comparison across criteria. Various criteria are usually measured in various units, the scores in the evaluation matrix have to be transformed to a normalized scale. The normalization of values can be carried out by one of the several known standardized formulas. The normalized value n_{ij} is calculated as follows:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}. \quad \text{(A3)}$$

Step 4. Consists of the calculations. Calculations will be made in order to weigh the normalized decision matrix. The weighted normalized value v_{ij} is calculated as follows:

$$v_{ij} = w_j n_{ij} \ for \ i = 1, \dots, m; j = 1, \dots, n, \quad (A4)$$

where:

w_j −the weight of the $j^{\text{-th}}$ criteria

Step 5. Includes the analysis and calculations of a positive ideal labeled as (V^+) and a negative ideal that is labeled as (V^-) solutions. The ideal positive solution is the solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria.

$$V^+ = (v_1^+, v_2^+, \dots, v_n^+) = \left((v_{ij} \mid j \in I), (v_{ij} \mid j \in J)\right) \quad (A5)$$

$$V^- = (v_1^-, v_2^-, \dots, v_n^-) = \left((v_{ij} \mid j \in I), (v_{ij} \mid j \in J)\right), \quad (A6)$$

where:

I is associated with benefit criteria and *J* with the cost criteria, $i = 1, \dots, m$; $j = 1, \dots, n$.

Step 6. Calculate the Euclidean distance from the ideal best (V^+) solution and the anti-ideal best (V^-) solution. The separation measures of each alternative from the ideal best (V^+) solution and the anti-ideal (V^-) solution, respectively, are as follows:

$$D_j^+ = \sqrt{\sum_{j=1}^{n} (vij - vj^+)^2}, \quad i=1,2,\dots,m \quad (A7)$$

$$D_j^- = \sqrt{\sum_{j=1}^{n} (vij - vj^-)^2}, \quad i=1,2,\dots,m \quad (A8)$$

Step 7. Calculate the relative closeness to the positive ideal solution. The relative closeness is defined as follows:

$$P_i = \frac{s_i^-}{s_i^- + s_i^+}, \quad (A9)$$

where:

$0 \leq P_i \leq 1$, $i = 1,2, \dots, m$.

Step 8. Rank the preference order.

To apply the TOPSIS method, all the steps described above should be performed and analyzed.