VILNIUS GEDIMINAS TECHNICAL UNIVERSITY

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**FLEXURAL BEHAVIOUR OF REINFORCED
CONCRETE MEMBERS STRENGTHENED
WITH PRESTRESSED CFRP LAMINATE**

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Abstract

The use of fibre reinforced polymer (FRP) materials for the strengthening of reinforced concrete structures has many benefits in comparison to the traditional strengthening methods. Such reinforcements are lighter, easier to install, they will better meet aesthetical design and anticorrosion/durability requirements. The full potential of such high strength materials can be exploited by prestressing. This way, the deflection of the element will be reduced and the concrete cracking and steel yield loads will be increased. However, the use of FRP's for strengthening may cause lots of issues for civil engineers regarding the appropriate evaluation of strengthened structures work, and technological issues during prestressing.

While the concrete member is uncracked, both elements deform together and it can be considered as a full composite action. However, after concrete cracking, stress concentrations appear at the cracks and it reduces the contact stiffness of concrete and FRP. Afterwards, the slip occurs in the contact surface, what leads to a sudden and brittle FRP debonding failure mode. These are the most common failure modes. Therefore, a number of studies have been conducted on debonding like failure modes. However, the failure criterion or fracture energy values in the models found in the literature were derived from experimental narrow parametric scope pull-off / pull-out shear tests, while the bond behaviour and this criterion could be different for strengthened flexural reinforced concrete members with long bond lengths. It should also be noted that the effect of concrete-FRP partial shear connection is usually ignored in the assessment of element deformations.

Considering the state of the art, a new analytical calculation model for the evaluation of concrete-FRP partial shear connection is proposed. Also, alternative new calculation techniques for the determination of the load carrying capacity, deflection, and cracking of the FRP strengthened RC members, taking into account FRP prestressing and concrete-FRP partial shear connection effects. Thus, the current thesis not only effectively evaluates the possible premature failure of the element, but also evaluates the full operation of concrete-FRP partial shear connection joint and its influence on the element deformations.

Furthermore, a novel prestressing system for externally bonded FRP laminates is proposed in response to the technological challenges of prestressing.

Reziumė

Pluoštu armuotų kompozitinių medžiagų panaudojimas gelžbetoninių konstrukcijų stiprinimui turi nemažai privalumų lyginant su tradiciniais metodais. Šios medžiagos yra lengviau tvirtinamos, turi geresnį estetinį vaizdą bei yra ilgaamžės. Šis kompozitas gali būti efektyviai panaudotas jį iš anksto įtempiant. Tokiu būdu ne tik padidinama stiprinamo elemento laikomoji galia, bet sumažinami įlinkiai ir padidinamas atsparumas pleišėjimui. Tačiau kompozitinių medžiagų panaudojimas konstrukcijų stiprinimui iškėlė nemažai naujų iššūkių statybos inžinieriams, susijusių su tinkamu konstrukcijos darbo vertinimu ir išankstinio įtempimo technologija.

Kol gelžbetoninė konstrukcija nesupleišėjusi betono ir kompozito jungtyje šlyties deformacijos nepasireiškia ir galima teigti, kad jungtis yra standi. Betone atsivėrus plyšiams, plyšio kraštuose susidaro įtempių koncentracija, tai sumažina betono ir kompozito jungties standumą, ir sukelia praslydimą, kuris iššaukia betono ir kompozito jungties trapų irimą. Atlikta analizė rodo, kad tokie irimo atvejai dažnai pasitaiko kompozitu sustiprintuose lenkiamuose elementuose. Tačiau literatūroje sutinkamuose modeliuose naudojamos irimo kriterijų ar irimo energijos reikšmės išvestos iš mažos parametrinės sklaidos kompozito atplėšimo bandymų, todėl jungties elgsena ir irimo kriterijai gali būti visai kitokie realaus dydžio lenkiamuose elementuose su gerokai didesniais kompozito tvirtinimo ilgiais. Dažnai skaičiuojant kompozitu sustiprintų gelžbetoninių elementų deformacijas ir pleišėjimą tarp betono ir kompozito vertinama standi jungtis.

Atsižvelgiant į gautus tyrimų rezultatus, pasiūlyti laikomosios galios, įlinkio, plyšio parametrų skaičiavimo metodai, įvertinant betono ir kompozito dalinai standžios jungties bei kompozito išankstinio įtempimo efektus. Disertaciniame darbe ne tik įvertinamas galimas priešlaikinis elemento irimas, bet ir aprašoma pilna dalinai standžios jungties tarp betono ir kompozito elgsena ir jos įtaka elemento deformacijoms ir pleišėjimui.

Atsižvelgus į technologinius anglies pluošto kompozito išankstinio įtempimo iššūkius, darbe pasiūlyta nauja kompozito išankstinio įtempimo sistema su efektyviais rifliuotais inkaravimo įrangos paviršiais.

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¹The annexes are supplied in the enclosed compact disc.

Introduction

Problem Formulation

The use of fibre reinforced polymer (FRP) materials has grown steadily in the strengthening of reinforced concrete structures in the last decades. There may be various reasons why the structure condition has deteriorated and strengthening is required, from design and construction mistakes to poor maintenance, corrosion, and ageing. The rational solutions for strengthening in most cases are FRP composites. Such reinforcements are lighter, easier to install in comparison to the traditional strengthening materials (e.g., concrete jackets, steel plates) what leads to the reduction of labour costs, and they will better meet aesthetical, design and anticorrosion/durability requirements. Moreover, the full potential of such materials can be exploited by prestressing. This way, the deflection of the element will be reduced and the concrete cracking and steel yield loads will be increased. However, all these benefits above can be corrupted by the loss of composite action of concrete and FRP material. While the structure is uncracked, both elements deform together and it can be considered as a full composite action. Nevertheless, after concrete cracking, stress concentrations appear at the cracks and it reduces the contact stiffness of RC and FRP. Afterwards, the slip occurs in the contact surface, what leads to a sudden and brittle FRP debonding failure mode. These are the most common failure modes. Therefore, a number of studies have been

conducted on debonding like failure modes. Generally, the existing bond strength models can be separated into six main groups: empirical models, shear capacity based models, concrete tooth models, interfacial stress based models, fracture energy based models, and intermediate crack induced (IC) debonding models. However, the failure criterion or fracture energy values in the previously mentioned models were derived from experimental narrow parametric scope pull-off / pull-out shear tests, while the bond behaviour and this criterion could be different for strengthened flexural reinforced concrete members with long bond lengths. It should also be noted that the effect of concrete-FRP partial shear connection is usually ignored in the analysis of the element deformations.

Relevance of the Thesis

Considering the state of the art, a new analytical calculation model for the evaluation of concrete-FRP partial shear connection is required. Also, the following new calculation techniques for the determination of the load carrying capacity, deflection, and cracking of reinforced concrete elements retrofitted with FRP, taking into account FRP prestressing and concrete-FRP partial shear connection effects. Thus, not only the possible premature failure of the element should be effectively evaluated, but also the full operation of concrete-FRP partial shear connection joint and its influence on the element deformations. Moreover, the behaviour of the beams and the deterioration of concrete-FRP joint of the RC beams strengthened under external load action were not properly studied yet. Although this is a common case in practise as usually the old structures under stress state require retrofitting. Therefore, the new experimental data on concrete-FRP partial shear connection and deformations of the element could influence further research on such structures and the practical application of composite materials.

In addition, the prestressing of FRP materials may cause lots of issues for civil engineers while strengthening existing structures under service load action. Therefore, the state of art and new anchorage solution is required.

The Object of Research

The object of the present study is concrete-FRP partial shear connection effect on cracking, deflection, and load-carrying capacity of reinforced concrete members externally strengthened with prestressed carbon fibre laminate and subjected to a static short-term load action.

The Aim of the Thesis

The aim is to develop a model evaluating the prestressing force and concrete-FRP partial shear connection effects on the cracking, deflection, and load-carrying capacity of reinforced concrete elements externally strengthened with prestressed carbon fibre composite.

The Tasks of the Thesis

To achieve the objective, the following problems had to be solved:

1. To develop calculation models for the evaluation of the load-carrying capacity, deflection, and crack parameters of the strengthened beams taking into account the effects of prestressing and partial shear connection between concrete and FRP.
2. To develop a device and technique for prestressing CFRP laminates externally bonded to RC elements.
3. To carry out experimental research of RC beams strengthened with prestressed CFRP laminates, and observe the change in deflection, strain and crack width.
4. To compare the experimental and theoretical results. Based on the obtained results, to refine the calculation methods.

Research Methodology

To investigate the object, the following research methods are chosen:

- Action: a theoretical model for evaluation of the load-carrying capacity, mid-span deflection, and crack width of RC members strengthened with prestressed FRP composites is developed.
- Experiment: an experimental study of RC beams strengthened with prestressed CFRP laminates is performed.
- Statistical analysis: conclusions are drawn collecting, analysing, and explaining the statistical data.

Scientific Novelty of the Thesis

The aspects of scientific novelty on theoretical and experimental investigation of reinforced concrete members strengthened with prestressed CFRP laminates are as follows:

1. A new analytical calculation model for the evaluation of concrete-FRP partial shear connection is proposed.
2. An alternative, new calculation models for the determination of the load carrying capacity, deflection, and cracking, taking into account FRP prestressing and concrete-FRP partial shear connection effects, are proposed.
3. A novel prestressing system with effective corrugated surfaces of the clamping unit and anchoring plates for externally bonded FRP laminates is proposed.
4. The new experimental data on the behaviour of concrete-FRP partial shear connection are obtained.
5. The new experimental results on RC beams externally strengthened with prestressed CFRP laminates and subjected to a static short-term load action, are obtained.

Practical Value of the Research Findings

In the present research, a proposed analytical calculation method for the evaluation of concrete-FRP partial shear connection considering the full degradation of the bond from concrete cracking to ultimate load. Also, calculation methods regarding load carrying capacity, deflection and cracking analysis of RC beams strengthened with prestressed FRP composites. A unique prestressing system for externally bonded FRP laminates with effective anchorage is proposed. The experimental program allows to investigate the partial shear connection of concrete and FRP, and FRP prestressing effects on the strengthened beam behaviour. Overall, theoretical and practical issues are solved regarding the appropriate evaluation of strengthened beam behaviour and technological issues of prestressing.

The Defended Statements

The following statements based on the results of present investigation may serve as the official hypotheses to be defended:

1. The proposed analytical calculation model for the evaluation of concrete-FRP partial shear connection adequately estimates the difference of concrete and FRP strain and the reduction of bond stiffness.
2. The proposed calculation models for load carrying capacity, deflection and cracking analysis considering concrete-FRP partial shear connection and FRP prestressing effects adequately assess the serviceability and ultimate states with respect to the concept of RC design.
3. Concrete crack parameters can be directly related to strains using the fracture mechanics based crack model for strengthened flexural members.
4. To obtain more accurate predictions of deflection and cracking, the concrete-FRP partial shear connection effect should be taken into account. Bond behaviour and tension stiffening are different for the beams strengthened under service loads.

Approval of the Research Findings

The results of the thesis have been published in 9 publications, 5 among them in the journals with an Impact Factor. The author has made 4 presentations at international and 1 national scientific conferences:

- 23rd International Conference on Composite Structures (ICCS23) & 6th International Conference on Mechanics of Composites (MECHCOMP6), Porto, Portugal.
- 5th International Conference on Mechanics of Composites (Mechcomp 5), Lisbon, Portugal.
- 13th International Conference „Modern Building Materials, Structures and Techniques“ (MBMST 2019), Vilnius, Lithuania.
- 9th International Conference on Fibre-Reinforced Polymer Composites in Civil Engineering (CICE 2018), Paris, France.
- 20th conference for Lithuanian Junior Researchers “Science-Future of Lithuania”, Civil Engineering, Vilnius, Lithuania.

Structure of the Dissertation

The dissertation is structured around three main chapters.

Chapter 1 reviews the calculation models regarding load bearing capacity, partial shear connection of concrete and FRP, deflection, crack width, and common prestressing devices for FRP strengthening systems. Chapter 1 concludes by formulating of the main objective and tasks of the present study.

Chapter 2 proposes improved prediction techniques of load bearing capacity, deflection, and crack width, taking into account prestressing and partial shear connection effects.

Chapter 3 presents the experimental investigation results on the cracking and deformation of reinforced concrete beams subjected to short-term load action, and a comparison of experimental and numerical results.

General conclusions summarise the present study. It is followed by an extensive list of references and a list of publications by the author on the topic of the dissertation.

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Review of the Behaviour Models and Failure Mechanisms of Fibre Reinforced Polymer Strengthened Reinforced Concrete Members

The first chapter presents a review of calculation models regarding load-carrying capacity, partial shear connection of concrete and FRP, deflection, and crack width of FRP strengthened RC members subjected to static short-term load action. The analysis is based on experimental and theoretical studies. Typical prestressing devices for FRP strengthening systems are also discussed. Finally, the first chapter concludes by formulating the aim and tasks of the present study. The review was partly presented in the articles published by Slaitas *et al.* (2017a, 2017b, 2018a, 2018b, 2018c, 2019, 2020, 2021a, 2021b).

1.1. Fibre Reinforced Polymer Strengthening Systems

The three common types of fibres are used in FRP systems: aramid (AFRP), carbon (CFRP), and glass fibres (GFRP) (Mcguirk and Brena, 2012, Bakis *et al.*

2002, Kim *et al.* 2014, ACI 440.2R-17, Teng *et al.* 2003). Despite the low cost of glass fibre reinforced polymer (GFRP), the more expensive carbon fibre reinforced polymer (CFRP) is the most appropriate FRP material for retrofitting existing structures for its higher strength and tensile modulus of elasticity close to steel. Typical stress-strain diagrams of unidirectional composites and steel subjected to short-term monotonic load action are presented in Fig. 1.1a (fib bulletin 14, 2001).

Fig. 1.1. Typical stress-strain diagrams: a) Uniaxial tension stress-strain diagrams for different unidirectional FRPs and steel (fib bulletin 14, 2001); b) Stress-strain relationship of fibres, matrix and FRP (CNR-DT 200 R1/2013)

The basic FRP strengthening technique involves the manual application of either wet lay-up or prefabricated systems (fib bulletin 14, 2001). In both cases, FRP composites consist of fibres and polymer matrix (see Fig. 1.1b). The basic mechanical properties of FRP can be found by applying “rule of mixtures” simplification as follows (fib bulletin 14, 2001):

$$E_f \approx E_{fib}V_{fib} + E_{mat}V_{mat}; \quad f_f \approx f_{fib}V_{fib} + f_{mat}V_{mat}, \quad (1.1)$$

where E_f , E_{fib} , E_{mat} are the moduli of elasticity (Young’s moduli) of FRP, fibres, and matrix, respectively; f_f , f_{fib} , f_{mat} are the tensile strength of FRP, fibres and matrix respectively; V_{fib} , V_{mat} are the volume fractions of fibres and matrix respectively.

FRP materials are considered linear elastic up to failure loads, i.e., they do not yield as steel, what can lead to sudden and brittle failure modes.

There are two main FRP systems regarding the type of reinforcement and the method of fastening:

- Externally bonded reinforcement (EBR);
- Near surface mounted reinforcement (NSM).

EBR means that FRP laminates (pre-manufactured) or sheets (wet lay-up system) are bonded to the external surface of the element (Fig. 1.2a). In the case of

NSM, grooves are cut on the concrete surface, into which premanufactured FRP strips (Fig. 1.2b) or rods (Fig. 1.2c) are glued.

Fig. 1.2. Main types of FRP composites used for strengthening:
a) EBR sheets/laminate; b) NSM strips; c) NSM rods

The main advantages of NSM technique over the EBR are found in the literature (De Lorenzis and Teng, 2007, Banijamali *et al.* 2015, Szabó and Balázs, 2007): (1) reduced labor costs on the construction site as there is no need for surface preparation, only grooving; (2) higher resistance to debonding - larger bond surfaces induce better anchorage capacity; (3) easier prestressing; (4) better protection of the reinforcement.

All above benefits can raise many questions. (1) Grooving is a more difficult and time-consuming process than surface grinding on a construction site (work at laboratory conditions may be different). Moreover, the damage is made on a concrete member and there is a high risk of cutting through the transverse reinforcement. In this respect, EBR seems to be a safer and easier to install option. (2) Bond surface of FRP composites (or the ratio of the bond surface and external reinforcement area) will depend on its cross-sectional dimensions. In both cases, the bond surface and the anchorage capacity may be better or worse. (3) The prestressing of NSM reinforcement is easier for a separate member at laboratory conditions, where the reinforcement can be anchored outside the element as in the case of precast prestressed concrete. However, the situation is much more complicated on the construction site. Since retrofitting is usually required for existing structures, the reinforcement anchorage outside the supports may be practically impossible. In this case, the NSM technique will have no advantage over the EBR. (4) The protection of FRP composites while applying NSM technique is better than EBR. However, given the fire requirements, additional protection measures will be required for FRP's in any case.

Whatever the method of strengthening is chosen, prestressing is required to exploit the potential of this high-strength material and improve the deformation properties of the existing structure. Generally, the prestressing of external FRP reinforcement can be processed via three systems: cambered beam system, tensioning against an independent beam system, tensioning against the strengthened

beam (El Hacha *et al.* 2001). The first system involves applying a vertical force and cambering the beam upward using large hydraulic jacks, bonding the FRP reinforcement into the design position, releasing and removing the jacks. Such a prestressing system does not require clamping and anchorage during prestressing, which causes a number of issues. However, a low level of prestress induced by the FRP reinforcement upon release of jacks is reported (Saadatmanesh and Ehsani, 1991, Char *et al.* 1994) after testing. The efficiency of the system is relatively low, assuming high costs.

The second technique involves an independent external prestressing system. In this case, the steps of prestressing are following: FRP reinforcement is bonded to the flat anchors, prestressed separately from the structure being strengthened, FRP is applied to the structure with prestressing rig and epoxy is cured, FRP is cut from the anchors and the prestress is transferred to the structure. Researchers (Deuring, 1993, Garden *et al.* 1998, Quantrill and Hollaway, 1998), who have tested the prestressing systems, reported sufficient levels of prestressing. Such a technique is convenient to use at laboratory conditions (see Fig. 1.3), where FRP reinforcement is prestressed to a rigid steel frame.

Fig. 1.3. Prestressing system for NSM CFRP reinforcement at laboratory conditions (Rezazadeh, 2015)

However, this kind of system, intended for a field application, as the one proposed by Barros (Rezazadeh *et al.* 2014), can only be used for members of limited length. It is a big downside in terms of a field application.

Fig. 1.4. Prestressing system for NSM CFRP strips for field application (Rezazadeh *et al.* 2014)

The third type of prestressing system is tensioning against the strengthened member. The FRP composites are directly tensioned by pulling on the anchors (or clamping unit), bonded to the FRP, and reacting on the anchors mounted on the strengthened member (El Hacha *et al.* 2001). The commercially available S&P Clever Reinforcement Company prestressing system uses aluminium plates for anchorage and clamping unit, which is pushed by a hydraulic jack mounted on the reaction frame (see Fig. 1.5).

Fig. 1.5. Prestressing system, courtesy of S&P Clever Reinforcement AG: a) Anchor plate during installation; b) reaction frame with hydraulic jack (Michels *et al.* 2016)

Full process of prestressing covers the following steps (Michels *et al.* 2016):

1. The surface is prepared to remove the concrete laitance and obtain a rough surface while minimising the exposure of aggregates (Fig. 1.6a).
2. Holes are drilled to accommodate the bolts to fix the aluminium anchor plates and base angles. The locations of these holes are previously marked on the element to be prestressed (Fig. 1.6b).
3. The clamp units are introduced between the base angles and cutting CFRP laminate to the desired length. The epoxy adhesive to bond the laminate to the concrete is then prepared and applied to the laminate. Finally, the laminate is inserted on the element to be strengthened between the clamp units (Fig. 1.6c-e).
4. The steel anchor, aluminium frame, and hydraulic jack are successively placed at both extremities (Fig. 1.6f-h).
5. The prestress is applied at the predefined load level (Fig. 1.6i).

Pretensioning is maintained by driving a set screw between a tensioning device and the clamping unit. The hydraulic jack is removed under retention of the pretensioning held by the screw. The tensioning device and the clamping unit are removed from the building after hardening of an adhesive (European patent EP2088259B1).

Fig. 1.6. Main steps for applying the prestress: a) surface preparation; b) drilling the holes for the bolts of the anchors and base angles; c) placing the clamp unit; d) introducing the laminate in the anchor; e) fixing the laminate in the anchor; f) placing the aluminium plate; g) placing the aluminium frame; h) placing the hydraulic jack; i) prestressing the laminate and controlling the deformation (Michels *et al.* 2016)

The main disadvantage is a risk of slipping in the contacting surfaces of the FRP laminate and the plane of aluminium block clamping the laminate.

Alternative prestressing system is presented by Sika (see Fig. 1.7), which uses pulling of the laminate instead of pushing. The system stands out for its clamping element made of soft and hard layers (stress-head). The clamping element comprises a structure without a wedge taper or a wedge-shaped or conical structure, where a cross-section reduction of the wedge runs against the tension direction of a tension element. The sleeve has an interior shape for retaining the clamping element and for exerting clamping pressure (European patent application EP2631392A1).

Fig. 1.7. Carbo stress prestressing system, courtesy of Sika Switzerland: a) prestressing system; b) stress-head anchorage for prestressed CFRP strips (Michels *et al.* 2016)

Despite the improved clamping/anchorage unit, the following issues can be found in the Sika system. The clamping unit is in tension, it means that the system will take place outside the end of the laminate, and even small eccentricities can cause longitudinal cracks in the laminate with the following tensile failure. Additionally, the end of the laminate will be moved away from the support, such a reduced ratio of the laminate and concrete element lengths can lead to the end debonding failure mode, especially for shorter concrete elements. Furthermore, this device requires additional damage to the concrete for using it in EBR strengthening systems. Longitudinal and transverse reinforcements will not let to make such a groove on a concrete surface in real structural members. The laminate will be too far from the concrete surface to bond it without a groove for the clamping unit.

One more alternative prestressing system was recently proposed by Siwowski (2020), which reduces the friction effect with rollers attached to the clamping unit (see Fig. 1.8). The thin high bond area steel plates are used for clamping and anchorage of the laminate, therefore a reliable methodology is required to calculate the anchorage area. The size of the plates should be adjustable accordingly, otherwise the reliability of the system will not be guaranteed.

Fig. 1.8. Prestressing system for EBR FRP laminates: a) scheme; b) prototype (Siwowski *et al.* 2020)

Prestressing systems for the NSM reinforcement work in a similar manner as EBR. For instance, the prestressing system proposed by El Hacha and Gaafar (2011) for NSM FRP rods uses a fixed steel anchor and a type of clamping unit – movable anchor and bracket, which is pushed with a hydraulic jack supported with a fixed bracket (see Fig. 1.9).

Fig. 1.9. Prestressing system for NSM CFRP bars: a) isometric view; b) elevation view at jacking end (El Hacha and Gaafar, 2011)

The review of prestressing devices has revealed the potential issues that may be encountered in prestressing FRP.

1.2. Load-Carrying Capacity and Full Shear Connection

Seven typical failure modes could be distinguished for RC structures strengthened with FRP (Smith and Teng, 2002, Teng *et al.* 2003): (1) flexural failure caused by FRP rupture; (2) flexural failure caused by crushing of compressive concrete; (3) shear failure; (4) separation of concrete cover of tensile reinforcement; (5) interfacial debonding in FRP plate end; (6) interfacial debonding caused by flexural crack in midspan of the beam; (7) interfacial debonding caused by intermediate flexural-shear crack. First two failure modes are the most desirable ones. Shear failure is out of the scope of current research. 4–7 are the undesirable sudden and brittle FRP debonding like failure modes discussed in the next chapter.

In the case of steel yield followed by concrete crushing or steel yield followed by FRP rupture (a sufficient level of ductility should be guaranteed), the strengthened cross section is calculated based on the principles of RC design (see Fig. 1.10).

Fig. 1.10. Internal strain and stress distribution for the rectangular section under flexure at ultimate limit state: a) RC section in EBR; b) RC section in NSM; c) strain distribution; d) force equilibrium (nonlinear concrete stress distribution); e) force equilibrium (equivalent concrete stress distribution) (based on ACI 440.2R-17, fib bulletin 14)

Two types of concrete compression stress diagrams can be used for the analysis: nonlinear (Fig. 1.10d) or equivalent (Fig. 1.10e). There are a number of techniques to describe concrete stress-strain relation and some of the most popular are Hognestad's (Hognestad, 1951), Popovics (Popovics, 1973), Carreira's and Chu's modified Popovics technique (Carreira and Chu, 1985), Eurocode (EN 1992-1-1:2004, Model Code 2010, STR 2.05.05:2005).

Second type of concrete compressive stress diagrams are the equivalent ones. Basis of the composition of the diagrams is presented in Fig. 1.11.

The equivalent factors of concrete compressive stress diagram:

$$\lambda = \frac{2x_{cc}}{x_c}; \quad \eta = \frac{A_{sum}}{f_c \lambda x_c}, \quad (1.2)$$

where x_{cc} is the coordinate of the centroid of the nonlinear concrete compressive zone stress diagram; x_c is a neutral axis depth; A_{sum} is the area of the nonlinear concrete compressive zone diagram, which can be found from the force equilibrium; f_c is the compressive strength of concrete.

Some values of the reduction factors for equivalent concrete stress diagram can be found in design codes (EN 1992-1-1:2004, Model Code 2010, fib bulletin 14, STR 2.05.05:2005, ACI 440.2R-17, ACI 318-14), or they can be calculated by the techniques based on Eurocode 2 nonlinear relation proposed by Dulinkas and Zabulionis (Dulinkas and Zabulionis 2007, Zabulionis and Dulinkas 2008) and 'ZI Method' (Židonis 2007, 2018, Tankūnas *et al.* 2012).

Equilibrium condition of forces (Fig. 1.10):

$$\eta f_c b \lambda x_c + A_{s2} \sigma_{s2} = A_{s1} \sigma_{s1} + A_f \sigma_f, \quad (1.3)$$

where A_{s1} , A_{s2} and A_f are the areas of tensile steel, compressive steel and external FRP reinforcement respectively.

Fig. 1.11. Stress distribution diagram for concrete in compression: a) strain in cross section; b) curvilinear concrete's compressive zone stress diagram and centroids of its parts: c – whole stress diagram, c_1 – ascending part, c_2 – constant (rectangular) part, c_3 – separated part from rectangular by curved descending line, x_c , x_{c1} , x_{c2} , x_{c3} – the coordinates of centroids of stress diagram parts respectively; c) rectangular stress diagrams: 3 – equivalent, 4 – inequivalent (Dulinskas *et al.* 2009)

Tensile steel, compressive steel and FRP reinforcement stress, respectively:

$$\sigma_{s1} = E_{s1} \varepsilon_{s1} \leq f_{s1,y}; \quad \sigma_{s2} = E_{s2} \varepsilon_{s2} \leq f_{s2,y}; \quad \sigma_f = E_f \varepsilon_f \leq f_f, \quad (1.4)$$

where E_{s1} , E_{s2} and E_f are the elastic modulus of tensile steel, compressive steel and external FRP reinforcements, respectively; ε_{s1} , ε_{s2} and ε_f are the strains of tensile steel, compressive steel and external FRP reinforcement respectively; $f_{s1,y}$, $f_{s2,y}$ and f_f are the yield stress of tensile steel, compressive steel and tensile strength of external FRP reinforcements respectively.

Bilinear elastic-plastic stress-strain relation is used for steel reinforcement.

Tensile steel and compressive steel reinforcement strains, respectively:

$$\varepsilon_{s1} = \varepsilon_c \frac{d - x_c}{x_c}; \quad \varepsilon_{s2} = \varepsilon_c \frac{x_c - d_2}{x_c}. \quad (1.5)$$

On a RC design basis, $\varepsilon_{s1} > f_{s1,y} / E_{s1}$ to ensure a sufficient level of ductility. FRP strain taking into account the initial strain state (ε_0) of the element (fib bulletin 14, ACI 440.2R-17):

$$\varepsilon_f = \varepsilon_c \frac{d_f - x_c}{x_c} - \varepsilon_0. \quad (1.6)$$

If the element is strengthened with prestressed FRP composites, the FRP strain intended for prestressing tendons could be used in the analysis (ACI 440.4R-04):

$$\varepsilon_f = \varepsilon_c \frac{d_f - x_c}{x_c} + \varepsilon_p, \quad (1.7)$$

where ε_p is the FRP prestressing strain, $\varepsilon_p = P / E_f$.

Prestressing force P is calculated taking into account the following losses (based on fib bulletin 14):

Initial losses:

- Immediate elastic deformation of the concrete that occurs when the prestress is transferred into the beam (Eq. 1.8).

$$\Delta P_{el} = \frac{E_f A_f P_0}{E_c A_c} \left(1 + \frac{e_p A_c}{I_c} \right), \quad (1.8)$$

where P_0 is the initial prestressing force; I_c is the second moment of area of the concrete section; A_c – cross-sectional area; e_p – eccentricity of prestressing force.

- Slip of the FRP reinforcement at the ends and deformation of the anchorage that occurs when the prestress is transferred into it. If the anchorage meets the requirements of fib bulletin 14, the loss of prestress can be estimated similarly to the pull-in of wedges as for steel tendons (Eq. 1.9).

$$\Delta P_a = E_f A_f \frac{\Delta_a}{L_f}, \quad (1.9)$$

where L_f is the length of FRP reinforcement between anchorages; Δ_a is the deformation of the anchorage, which can be taken equal to 2 mm.

Time-dependent losses:

- Relaxation of the prestressed FRP reinforcement. In accordance with Model Code 2010, the relaxation of GFRP, CFRP and ARFP prestressing elements after 50 years of loading can be estimated as 4–14%, 2–10% and 11–25% respectively. These values depend on the stress level and environmental influence. Guidelines given in Model Code 2010 should be used for more accurate analysis.
- Creep and shrinkage of the concrete. Since strengthening is generally applied to existing old structures, the loss of prestress due to shrinkage of concrete in this case will be negligible. On the other hand, the creep of concrete must be taken into account in all cases. Loss of prestress corresponding to time-dependent factors in accordance with Eurocode 2 can be expressed by the following equation.

$$\Delta P_{c+s+r} = A_f \frac{\varepsilon_{cs} E_f + 0.8 \Delta \sigma_{f,r} + E_f \varphi(t, t_0) \sigma_{c,QP} / E_c}{1 + E_f A_f (1 + A_c d^2 / I_c) (1 + 0.8 \varphi(t, t_0)) / E_c A_c}, \quad (1.10)$$

where ε_{cs} is the estimated shrinkage strain according to Eurocode 2 in absolute value; $\Delta \sigma_{f,r}$ is the absolute value of loss of prestress due to relaxation of FRP; $\varphi(t, t_0)$ is the creep coefficient at time t and post-tensioning load application time t_0 ; $\sigma_{c,QP}$ is the stress in the concrete adjacent to FRP reinforcement, due to self-weight, initial prestress and other quasi-permanent actions where relevant; a is the distance between centre of gravity of the concrete and the FRP reinforcement.

Afterwards, post-tensioning force will be as follows:

$$P = P_0 - \Delta P_{el} - \Delta P_a - \Delta P_{c+s+r}. \quad (1.11)$$

If there are any thermal deformations, they should be taken into account.

Combining equations 1.6 and 1.7 gives the following full expression of FRP strain:

$$\varepsilon_f = \varepsilon_c \frac{d_f - x_c}{x_c} - \varepsilon_0 + \varepsilon_p. \quad (1.12)$$

There are two possible cases: concrete crushing controlled failure mode and tensile failure controlled failure mode. The first case: $\sigma_f \leq f_f$ and $\varepsilon_c = \varepsilon_{cu}$ (the compressive strain of concrete is equal to the ultimate value) and the second one: $\sigma_f > f_f$ and $\varepsilon_c < \varepsilon_{cu}$. If the element failure is controlled by tensile zone failure, when FRP stress $\sigma_f = f_f$ and concrete strain is found by the following equation (based on fib bulletin 14 and assumptions above):

$$\varepsilon_c = \left(\frac{f_f}{E_f} + \varepsilon_0 - \varepsilon_p \right) \frac{x_c}{d_f - x_c}. \quad (1.13)$$

Finally, the ultimate bending moment can be expressed by the bending moment equilibrium condition. The sum of the bending moments is usually calculated around the resultant points of tensile steel reinforcement (Eq. 1.14) or the compressive zone of concrete.

$$M_u = \eta f_c b \lambda x_c \left(d - \frac{\lambda x_c}{2} \right) + A_f \sigma_f (d_f - d) + A_{s2} \sigma_{s2} (d - d_2). \quad (1.14)$$

American design codes (ACI 440.2R-17) add two additional factors to load-carrying capacity. An additional reduction factor for FRP, ψ_f , is applied to the flexural strength contribution of the FRP reinforcement. Additionally, the strength reduction factor, ϕ , is applied to account for a less ductile failure. Afterwards, the ultimate bending moment gets the following expression.

$$M_u = \phi \left(A_{s1} \sigma_{s1} \left(d - \frac{\lambda x_c}{2} \right) + \psi_f A_f \sigma_f \left(d_f - \frac{\lambda x_c}{2} \right) + A_{s2} \sigma_{s2} \left(\frac{\lambda x_c}{2} - d_2 \right) \right). \quad (1.15)$$

The recommended value of ψ_f is 0.85 (ACI 440.2R-17) and for ϕ (ACI 440.2R-17, ACI 318-14):

$$\phi = \begin{cases} 0.90, & \varepsilon_{s1} \geq 0.005, \\ 0.65 + \frac{0.25(\varepsilon_{s1} - \varepsilon_{s1,y})}{0.005 - \varepsilon_{s1,y}}, & \varepsilon_{s1,y} < \varepsilon_{s1} < 0.005, \\ 0.65, & \varepsilon_{s1} \leq \varepsilon_{s1,y}, \end{cases} \quad (1.16)$$

where $\varepsilon_{s1,y} = f_{s1,y} / E_{s1}$.

It can be concluded that the current chapter has provided a full methodology for the evaluation of the load carrying capacity of RC-FRP composite elements with full shear connection.

1.3. Bond Strength and Partial Shear Connection

1.3.1. Empirical Models

Load-carrying capacity evaluation methodology is described in a previous section for FRP strengthened RC members in bending with full shear connection. However, after concrete cracking, stress concentrations appear at the cracks and the slip occurs in a concrete-FRP contact surface, what leads to the loss of composite

action and sudden and brittle FRP debonding failure modes. Therefore, FRP-concrete bond strength should be checked in parallel with the methodology presented in the previous chapter. Generally, bond strength models can be separated into 6 main groups: empirical models, shear capacity based models, concrete tooth models, interfacial stress based models, fracture energy based models, and intermediate crack induced (IC) debonding models.

Basic pull-off shear test and beam loading schemes are presented in Figures 1.12 and 1.13, respectively.

Fig. 1.12. Pull-off shear test schemes: a) EBR; b) NSM

Fig. 1.13. Beam loading schemes: a) 4 point bending; b) uniform load

Experimental pull-off / pull-out shear test results (Bilotta *et al.* 2016, Yao *et al.* 2005a, 2005b, Seracino *et al.* 2007, Carrara *et al.* 2011, Zhang *et al.* 2013, van Gemert, 1980, Oehlers and Moran, 1990, Täljsten, 1994, Malek *et al.* 1998, Sato *et al.* 2001, Yuan *et al.* 2004, Ferracuti *et al.* 2007) have shown that the stress in FRP reinforcement has an exponential form (Fig. 1.14a) or in higher bond length cases a parabola exponential form (Fig. 1.14b).

Models based on simple assumptions and calibrated with experimental data (van Gemert, 1980, Sato *et al.* 2001, Hiroyuki and Wu, 1997, Maeda *et al.* 1997, Khalifa *et al.* 1998, Izumo *et al.* 1999, Yang *et al.* 2001, JCI, 2003) could be called empirical or semi-empirical.

Fig. 1.14. Shear stress dependence on distance from the free end: a) exponential; b) parabola exponential

The basic equation used for ultimate load:

$$F_u = \tau_u A_b = \tau_u L_b b_f, \quad (1.17)$$

where τ_u is the ultimate shear stress, L_b and b_f are the bond length and width, respectively.

Models mentioned above have a theoretical basis, but the one proposed in U.S. design code (ACI 440.2R-17) for NSM FRP reinforcement should be called fully empirical as it is proposed to use 0.7 factor for the calculated strain. Such an assumption was based on Bianco *et al.* (Bianco *et al.* 2014) experimental database. Another example of a fully empirical model is Seracino *et al.* (Seracino *et al.* 2007) model derived from regression analysis for NSM FRP strips.

1.3.2. Shear Capacity Based Models

Shear capacity based models take the assumption that FRP end debonding occurs when the shear capacity of concrete is reached neglecting shear reinforcement (Oehlers and Moran, 1990, Oehlers, 1992, Jansze, 1997, Matthys, 2000, Colotti *et al.* 2004, 2006) or taking into account the partial effect of it (Ziraba, 1993, Ziraba *et al.* 1994, 1995, Ahmed and van Gemert, 1998, 1999).

General equations for shear capacity:

$$V_R = V_c + k_V V_s; \quad (1.18)$$

$$V_R = \tau_u d b_f, \quad (1.19)$$

where V_c and V_s are the shear capacity of concrete and transverse reinforcement, respectfully, k_V is the contribution factor of transverse reinforcement.

1.3.3. Concrete Tooth Models

Concrete tooth model was presented by Kani (Kani, 1964) for reinforced concrete element shear capacity. The basic idea of this method is to consider ‘concrete teeth’ between adjacent cracks as cantilevers fixed to the uncracked arch. Based on this concept, Raoof and Zhang (Zhang *et al.* 1995, Raoof and Zhang, 1997) proposed a model for predicting concrete cover separation of beams strengthened with soffit plates. Debonding occurs when the tensile stresses at the root of the ‘tooth’ (point A in Fig. 1.15) exceed the tensile strength of concrete.

Fig. 1.15. Concrete tooth model: a) cracked beam; b) concrete tooth between two adjacent flexural cracks (Smith and Teng, 2002)

General equation for this model:

$$\sigma_A = \frac{M_A L_{cr}}{2 I_A}, \quad (1.20)$$

where $M_A = \tau_f L_{cr} b_f c$ and $I_A = b_f L_{cr}^3 / 12$, c is the depth of concrete cover.

1.3.4. Interfacial Stress Based Models

There are two common approaches for interfacial stress models: principle stress model (Malek *et al.* 1998, Saadatmanesh and Malek, 1998, Tumialan *et al.* 1999), and Mohr-Coulomb's failure criterion (Ziraba, 1993, Ziraba *et al.* 1994, 1995, Varastehpour and Hamelin, 1997, Chaallal *et al.* 1998, Hassan and Rizkalla, 2003). In the first approach, debonding failure occurs when the principle concrete stress at the end of the FRP reinforcement reaches tensile strength (see Fig. 1.16). Governing equation:

$$\sigma_{end} = \left(\frac{\sigma_{z,end} + \sigma_{y,end}}{2} \right) + \sqrt{\left(\frac{\sigma_{z,end} + \sigma_{y,end}}{2} \right)^2 + \tau_{end}^2} \leq f_{ct}. \quad (1.21)$$

Fig. 1.16. Stresses acting on a concrete element adjacent to the plate end (Smith and Teng, 2002, Saadatmanesh and Malek, 1998, Tumialan *et al.* 1999)

Second approach is based on a set of linear equations in the principal stress space describing the conditions for which an isotropic material will fail (Fig. 1.17) (Labuz and Zang, 2012).

Fig. 1.17. Mohr-Coulomb failure criterion (Hassan and Rizkalla, 2003, Labuz and Zang, 2012)

Coulomb's condition is based on a linear failure envelope to determine the critical combination of σ , τ that will cause failure on some planes (Labuz and Zang, 2012). The governing equation for this approach proposed by Coulomb in his investigation of retaining walls:

$$\tau_{E.end} + \sigma_{E.end} \tan \varphi \leq C_{coh}, \quad (1.22)$$

where C_{coh} is the inherent shear strength, also known as cohesion, and φ is the angle of internal friction, with the coefficient of internal friction $\tan \varphi$ (Labuz and Zang, 2012).

The failure criterion or cohesion in the current approach is empirical, derived from the pull-off (pull-out) shear test results.

1.3.5. Fracture Energy Based Models

Bond strength models based on fracture mechanics are the most popular (Bilotta *et al.* 2011, Zhang *et al.* 2013, Täljsten, 1994, Yuan *et al.* 2004, Lu *et al.* 2005, Dai *et al.* 2005, Seracino *et al.* 2007, De Lorenzis and Teng, 2007, CNR-DT 200 R1/2013, Zhang, 2012, Zhang *et al.* 2014, Yuan and Wu, 1999, Holzenkämpfer, 1994, Blaschko *et al.* 1998, Neubauer and Rostásy, 1997, 1999, Niedermeier, 2000). The science of fracture mechanics of solids dates to the 1920's when Griffith (Griffith, 1921) formulated the energy balance condition for cracked brittle bodies. However, the highest impact was made by the works of Orowan and Irwin in 1949 – 1962 (Jokūbaitis, 2001) when the consideration of plastic deformation at the crack tip was proposed. The three main fracture or standard loading modes of a crack were analysed by Irwin (Irwin, 1958, Tada *et al.* 2000, Sih *et al.* 1965): opening mode – a tensile stress normal to the plane of the crack (mode I), sliding mode – a shear stress acting parallel to the plane of the crack and perpendicular to the crack front (mode II), tearing mode – a shear stress acting parallel to the plane of the crack and parallel to the crack front (mode III) (Fig. 1.18).

Fig. 1.18. Fracture modes

Mode II or mixed mode (I and II) fracture energies are the most common for the description of concrete-FRP bond behaviour. Generally, the mode II fracture energy will be the area of the bond shear stress-slip relation graph (see Fig. 1.19).

To describe the nonlinear bond shear stress – slip relation (Fig. 1.19a), Nakaba *et al.* (Nakaba *et al.* 2001) applied Popovics (Popovics, 1973) model of nonlinear concrete stress – strain relationship for EBR FRP laminates:

$$\tau_f(s_f) = \tau_{fm} \frac{s_f}{s_{fm}} \frac{n}{(n-1) + (s_f/s_{fm})^n}. \quad (1.23)$$

Fig. 1.19. Bond shear stress-slip models: a) nonlinear; b) linear ascending; c) linear descending; d) exponential; e) bilinear; f) multilinear

Lu *et al.* (Lu *et al.* 2005) proposed the following nonlinear relation for EBR:

$$\tau_f(s_f) = \begin{cases} \tau_{fm} \left[\sqrt{\frac{s_f}{s_{fm} - \tau_{fm}/K_b} + \left(\frac{\tau_{fm}/K_b}{2(s_{fm} - \tau_{fm}/K_b)} \right)^2} - \frac{\tau_{fm}/K_b}{2(s_{fm} - \tau_{fm}/K_b)} \right], & s_f \leq s_{fm}; \\ \tau_{fm} \exp(-n(s_f/s_{fm} - 1)), & s_f > s_{fm}, \end{cases} \quad (1.24)$$

where K_b is the initial bond stiffness.

On the other side, Barros and Sena Cruz (Barros and Sena Cruz, 2004) derived the power function from NSM CFRP strip pull-out bending shear test results:

$$\tau_f(s_f) = \begin{cases} \tau_{fm} (s_f/s_{fm})^n, & s_f \leq s_{fm}; \\ \tau_{fm} (s_f/s_{fm})^{-n'}, & s_f > s_{fm}. \end{cases} \quad (1.25)$$

Linear ascending relation (Yuan *et al.* 2001) (Fig. 1.19b):

$$\tau_f(s_f) = \frac{\tau_{fm}^2}{2G_f} s_f. \quad (1.26)$$

Linear descending relation (Yuan *et al.* 2001) (Fig. 1.19c):

$$\tau_f(s_f) = \tau_{fm} \left(1 - \frac{\tau_{fm}}{2G_f} s_f \right). \quad (1.27)$$

Or (Chen *et al.* 2007):

$$\tau_f(s_f) = \frac{\tau_{fm}}{s_{fm}} (s_{fm} - s_f). \quad (1.28)$$

Exponential relation (Yuan *et al.* 2001) (Fig. 1.19d):

$$\tau_f(s_f) = \tau_{fm} \exp \left(-\frac{\tau_{fm}}{G_f} s_f \right). \quad (1.29)$$

Bilinear relation (Yuan *et al.* 2001, 2004, Teng *et al.* 2006) (Fig. 1.19e):

$$\tau_f(s_f) = \begin{cases} \frac{\tau_{fm}}{s_{fm}} s_f, & s_f \leq s_{fm}; \\ \frac{\tau_{fm}}{s_{f\max} - s_{fm}} (s_{f\max} - s_f), & s_f > s_{fm}. \end{cases} \quad (1.30)$$

Multilinear relation (Bianco *et al.* 2007) (Fig. 1.19f):

$$\tau_f(s_f) = \begin{cases} \tau_0 + \frac{\tau_{fm} - \tau_0}{s_{fm}} s_f, & s_f \leq s_{fm}; \\ \tau_{fm} - \frac{\tau_{fm} - \tau_{f2}}{s_{f2} - s_{fm}} (s_f - s_{fm}), & s_{fm} < s_f \leq s_{f2}; \\ \tau_{f2} - \frac{\tau_{f2}}{s_{f\max} - s_{f2}} (s_f - s_{f2}), & s_{f2} < s_f \leq s_{f\max}. \end{cases} \quad (1.31)$$

Of course, there are other modifications of the multilinear relation as the one proposed by Mazaheripour (Mazaheripour, 2015) in Fig. 1.20.

Fig. 1.20. Multilinear bond shear stress-slip model (Mazaheripour, 2015)

In his thesis (Mazaheripour, 2015), Mazaheripour tried to improve Bianco's (Bianco *et al.* 2007) multilinear relation and more precisely repeated nonlinear graph. He has separated four main phases: elastic, plastic, softening and frictional (Fig. 1.20):

$$\tau_f(s_f) = \begin{cases} \tau_0 + \frac{\tau_{fm} - \tau_0}{s_{f1}} s_f, & s_f \leq s_{f1}; \\ \tau_{fm}, & s_{f1} < s_f \leq s_{f2}; \\ \tau_{fm} - \frac{\tau_{fm} - \tau_{f2}}{s_{f3} - s_{f2}} (s_f - s_{f2}), & s_{f2} < s_f \leq s_{f3}; \\ \tau_{f2}, & s_f > s_{f3}. \end{cases} \quad (1.32)$$

Moreover, possible different combinations of the stress-slip relations in Fig. 1.19, for instance, Cornetti and Carpinteri (Cornetti and Carpinteri, 2011) combined the linear ascending part with the exponential descending softening part:

$$\tau_f(s_f) = \begin{cases} \frac{\tau_{fm}}{s_{fm}} s_f, & s_f \leq s_{fm}; \\ \tau_{fm} \exp(-2n^2 (s_f/s_{fm} - 1)), & s_f > s_{fm}. \end{cases} \quad (1.33)$$

Bond shear stress and slip values in Eqs 1.23-1.33 can be found using linear or nonlinear fracture mechanics and basic interfacial stress laws. However, all calculations bounce back to the fracture energy, which is usually derived from pull-out/pull-off shear test results.

1.3.6. Intermediate Crack Induced Debonding Models

The intermediate crack-induced (IC) debonding failure modes were extensively analysed in the last decades as it was discovered that such a debonding mode is more common for the members strengthened with relatively thin FRP composites and the methods derived for the members strengthened with much greater stiffness steel plates were found unsatisfactory (Smith and Teng, 2002a, 2002b, Teng *et al.* 2003). Due to the reasons above, the design codes (ACI 440.2R-17, CNR-DT 200 R1/2013, JSCE, 2001, fib bulletin 14, 2001, Model Code 2010) adopted the existing models and went on the fracture energy and IC debonding way.

IC debonding occurs when horizontal cracks form at the stress concentration points on the tips of flexural cracks (Fig. 1.21).

Fig. 1.21. IC debonding failure mode (Teng *et al.* 2003)

Current methods describe the bond behaviour between two adjacent flexural cracks (see Fig. 1.22).

However, the existing IC debonding methods are based on the same laws of fracture mechanics and once again all calculations bounce back to the fracture energy, which is usually derived from pull-out/pull-off shear test results.

Fig. 1.22. Concrete-FRP joint between two adjacent flexural cracks (Chen *et al.* 2007)

The Chen and Teng's model (Chen and Teng, 2001) is distinguished in the literature (Lu *et al.* 2005) as having the highest level of accuracy in predicting debonding failure of pull-off shear tests. However, this technique is valid only for unanchored EBR systems, and other methods presented above have also many restrictions in use. Moreover, the research of Toutanji *et al.* (Toutanji *et al.* 2007) has shown that even Chen and Teng's model (Chen and Teng, 2001) can have high errors (of course on the safety side) when the test parameters are changed. Furthermore, the models derived from pull-off shear tests do not take into account the concrete block depth, which, according to (Yao *et al.* 2005) pull-push shear test has influence on the strength of the joint. Finally, the failure criterion or fracture energy was derived from experimental narrow parametric scope pull-off / pull-out shear tests, while the bond behaviour and this criterions could be different for strengthened flexural reinforced concrete members with long bond lengths.

1.3.7. Built-Up Bars Theory

The universal built-up bars theory (Rzhanitsyn, 1986, Valivonis and Marčiukaitis, 2006, Marchukaitis *et al.* 2007, Skuturna, 2009, Skuturna and Valivonis, 2014) is distinguished from the common bond strength models discussed above as it had a major influence on the models proposed in the current paper. Based on this theory, the joint of two separate members is being considered (Fig. 1.23) and the slip between them will be the difference of displacement (Eq. 1.34).

$$s = u_f - u_c. \quad (1.34)$$

Fig. 1.23. Bond model of concrete and FRP members

The first derivative of the Eq. 1.34 will be the difference of strains.

$$\frac{ds}{dz} = \varepsilon_f - \varepsilon_c = \frac{T}{E_f A_f} - \frac{T}{E_c A_{c,eff}} - \frac{M}{EI} a. \quad (1.35)$$

Overall bending moment in Eq. 1.35:

$$M = \sum M_i - Ta. \quad (1.36)$$

Then the rearranged Eq. 1.36:

$$\frac{ds}{dz} = \left(\frac{1}{E_f A_f} + \frac{1}{E_c A_{c,eff}} + \frac{a^2}{EI} \right) T - \frac{\sum M_i}{EI} a, \quad (1.37)$$

where the effective concrete area is $A_{c,eff} = bx_c$ and the total flexural stiffness of the two elements is:

$$EI = E_c I_{c,eff} + E_f I_f \approx E_c I_{c,eff} = E_c \frac{bx_c^3}{3}. \quad (1.38)$$

Generally, the material's response to the shear stress is described by the shear modulus in material science:

$$G_{eff} = \frac{\tau_{zy}}{\gamma_{zy}} = \frac{\tau_{zy}}{\Delta z / a} \approx \frac{\tau_{zy}}{s / a} \Rightarrow s = \frac{\tau_{zy} a}{G_{eff}} = \frac{a}{G_{eff} u_f} \frac{dT}{dz}. \quad (1.39)$$

Next, the following differential equation is obtained by aligning the derivative of Eq. 1.39 with Eq. 1.37.

$$\frac{a}{G_{eff} u_f} \frac{d^2 T}{dz^2} = \left(\frac{1}{E_f A_f} + \frac{1}{E_c A_{c,eff}} + \frac{a^2}{EI} \right) T - \frac{\sum M_i}{EI} a. \quad (1.40)$$

The following auxiliary quantities are introduced to simplify the equations:

$$\alpha = \frac{G_{eff} u_f}{a}; \quad \beta = \frac{1}{E_f A_f} + \frac{1}{E_c A_{c,eff}} + \frac{a^2}{EI}. \quad (1.41)$$

Afterwards, the second derivative of Eq. 1.40 will be as follows:

$$\frac{d^4 T}{dz^4} - \alpha \beta \frac{d^2 T}{dz^2} - \frac{\alpha p}{EI} a = 0. \quad (1.42)$$

The solution of Eq. 1.42 depends on the load conditions of the element. The following is the solution when the simply supported element is loaded with two concentrated forces $F/2$:

$$T(z) = \frac{Faz}{\beta EI} \left(1 - \frac{\cosh(\sqrt{\alpha\beta}(0.5L - L_s))}{\sqrt{\alpha\beta}z \cosh(0.5\sqrt{\alpha\beta}L)} \sinh(\sqrt{\alpha\beta}z) \right). \quad (1.43)$$

For a uniformly distributed load p :

$$T(z) = \frac{pa(L-z)z}{2\beta EI} \left(1 - \frac{2(\cosh(0.5\sqrt{\alpha\beta}L) - \cosh(\sqrt{\alpha\beta}(0.5L - z)))}{\alpha\beta z(L-z) \cosh(0.5\sqrt{\alpha\beta}L)} \right). \quad (1.44)$$

The first member in equations 1.43 and 1.44 represents the shear force of two rigidly connected members and the second one in brackets is the contact stiffness reduction factor ψ_f . For the beam loaded with two concentrated forces:

$$\psi_f = 1 - \frac{\cosh(\sqrt{\alpha\beta}(0.5L - L_s))}{\sqrt{\alpha\beta}L_s \cosh(0.5\sqrt{\alpha\beta}L)} \sinh(\sqrt{\alpha\beta}L_s). \quad (1.45)$$

concrete in resistance to tensile stress is referred to as tension-stiffening effect. To assess this effect, zones with different stiffness can be changed into simple springs. Examples of linear and rotational springs arranged in series and parallel are presented in Fig. 1.25.

Fig. 1.25. Simple spring model: springs in series: a) linear; b) rotational; springs in parallel: c) linear; d) rotational (Bischoff and Scanlon, 2007)

Applying equilibrium and compatibility to the linear elastic systems gives the following expressions for the effective stiffness of the two spring models considered (Bischoff and Scanlon, 2007). Springs in series:

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}. \quad (1.47)$$

Springs in parallel:

$$k_e = k_1 + k_2, \quad (1.48)$$

where k_1 , k_2 , k_e – stiffness of the springs representing cracked, uncracked zones and overall effective stiffness. P represents the axial and rotational force and Δ – linear and rotational displacement in Fig. 1.25.

Branson's original formulation (Branson, 1963) adopted by U.S. design codes before 2019 (ACI 318-14, ACI 435R-95, ACI 209R-92) is based on springs in parallel, on the other hand Bischoff (Bischoff and Scanlon, 2007, Bischoff, 2005) proves that springs in series are more appropriate for members with discrete cracks along the members. Such an approach of spring in series is used in European and International design standards (Eurocode 2, Model Code 2010, fib bulletin 14) and U.S. design code (ACI 318-19) after 2019 as well. The change of an

effect of uncracked zone resistance can be evaluated by the ratio of concrete cracking moment (M_{cr}) and acting moment (M_a). Then the effective stiffness of the element based on the original Branson's approach adopted by U.S. design codes before 2019 (ACI 318-14, ACI 435R-95, ACI 209R-92):

$$E_c I_e = E_c \left(\left(\frac{M_{cr}}{M_a} \right)^m I_g + \left(1 - \left(\frac{M_{cr}}{M_a} \right)^m \right) I_{cr} \right) \leq E_c I_g, \quad (1.49)$$

where the recommended value of power m is 3.0. Second moment of the cracked rectangular cross-section area, with the allusion to springs in parallel (see Fig. 1.26):

$$I_{cr} = \frac{b x_c^3}{3} + \alpha_{s1} A_{s1} (d - x_c)^2 + (\alpha_{s2} - 1) A_{s2} (x_c - d_2)^2 + \alpha_f A_f (d_f - x_c)^2, \quad (1.50)$$

where $\alpha_{s1} = E_{s1} / E_c$, $\alpha_{s2} = E_{s2} / E_c$, $\alpha_f = E_f / E_c$.

Fig. 1.26. Cracked cross-section modelling with springs in parallel

Bischoff's (Bischoff and Scanlon, 2007, Bischoff, 2005) approach (springs in series):

$$E \frac{1}{E_c I_e} = \frac{1}{E_c} \left(\left(\frac{M_{cr}}{M_a} \right)^m \frac{1}{I_g} + \left(1 - \left(\frac{M_{cr}}{M_a} \right)^m \right) \frac{1}{I_{cr}} \right) \geq \frac{1}{E_c I_g}; \quad (1.51)$$

$$E_c I_e = \frac{E_c I_{cr}}{1 - \left(\frac{M_{cr}}{M_a} \right)^m \left(1 - \frac{I_{cr}}{I_g} \right)} \leq E_c I_g. \quad (1.52)$$

Afterwards, equivalent curvature of the element by Bischoff's (Bischoff and Scanlon, 2007, Bischoff, 2005) approach:

$$\kappa_e = \frac{M_a}{E_c I_e} = \frac{M_a \left(1 - \left(\frac{M_{cr}}{M_a} \right)^m \left(1 - \frac{I_{cr}}{I_g} \right) \right)}{E_c I_{cr}} = \quad (1.53)$$

$$\frac{M_a}{E_c I_{cr}} \left(1 - \left(\frac{M_{cr}}{M_a} \right)^m \right) + \frac{M_a}{E_c I_g} \left(\frac{M_{cr}}{M_a} \right)^m = \kappa_{cr} \left(1 - \left(\frac{M_{cr}}{M_a} \right)^m \right) + \kappa_g \left(\frac{M_{cr}}{M_a} \right)^m.$$

Consequently, the expression of equivalent curvature is the same as it is recommended in European and International design codes (Eurocode 2, Model Code 2010, fib bulletin 14). The current version of U.S. design code (ACI 318-19) suggests to use $2/3 M_{cr}$ instead M_{cr} to consider a restraint that can reduce the effective cracking moment as well as to account for the reduced tensile strength of concrete during construction that can lead to cracking that later affects service deflections (Scanlon and Bischoff 2008).

The recommended value of power m in this approach is 2.0. Equations (1.49–1.53) above are valid for short-term loading, while the internal reinforcement is deformed steel bars. Otherwise, additional factors should be added to the tension stiffening effect.

Concrete cracking moment in the equations above can be found from the general linear elastic structural analysis for members exposed to bending moment and axial force.

$$\sigma = \frac{M}{I_g} y_t + \frac{N}{A_c} \Rightarrow M_{cr} = \frac{f_{ct} I_g}{y_t} + P \left(e_p + \frac{I_g}{A_c y_t} \right). \quad (1.54)$$

Same way, concrete cracking moment is derived by U.S., European and International design codes (Eurocode 2, fib bulletin 14, ACI 318-14, ACI 440.2R-17).

Second member in Eq. 1.54 is the secondary bending moment due to prestress corresponding to the deflection $\delta = 0$, elsewhere (Atutis *et al.* 2015, Atutis, 2018) so-called decompression moment.

$$M_p = P \left(e_p + \frac{I_g}{A_c y_t} \right), \quad (1.55)$$

where y_t – distance from the extreme bottom fibre to the section centroid; A_c – cross-sectional area; e_p – eccentricity of prestressing force.

Governing differential equations for the beam deflection in mechanics of solids:

$$EI \frac{dy}{dz} = EI\kappa; \quad (1.56)$$

$$EI \frac{d^2 y}{dz^2} = EI \frac{d\kappa}{dz} = -M. \quad (1.57)$$

Deflection from Eq. 1.57:

$$\delta = -\frac{1}{EI} \iint M dz. \quad (1.58)$$

General solution of Eq. 1.58 used in design codes (Eurocode 2, Model Code 2010, fib bulletin 14, ACI 318-14, ACI 435R-95, ACI 209R-92) will be as follows.

$$\delta = \xi \kappa L^2. \quad (1.59)$$

where ξ – load adding coefficient (four point bending case: $\xi = 1/8 - (L_s/L)^2/6$).

Matthys (2000) proposed to use linear elastic analysis of the cracked section assuming linear elastic material behaviour and that the concrete does not sustain tension. Such an approach was adopted by fib bulletin 14. The difference from the classical RC design is that the neutral axis depth becomes dependent on the acting bending moment as a result of the initial strain before strengthening.

As RC analysis methods are also used for the analysis of strengthened structures, alternative methods for the assessment of tension stiffening effect should be mentioned. One of these is the division of the structure into layers (Fig. 1.27), afterwards, nonlinear material stress diagrams are used for the iterative calculation procedure of each layer (Kaklauskas 2004, 2011). The full stress diagram of the tensile concrete is used in the tension stiffening part.

Fig. 1.27. Layer section model: a) RC section; b) layered section; c) stress distribution across the section; d) and e) i^{th} layer strain and stress (Kaklauskas *et al.* 2011)

Another approach is the direct integration of the curvature graph – solution of Eq. 1.56. In this case, a simply supported beam is divided in distinct regions along the length corresponding to these governing stages, namely, precracking, postcracking and postyielding regions, as shown in Fig. 1.28a (Rezazadeh *et al.* 2015). The curvature of different regions is found from the strain profile (Fig. 1.28b). When the solution of Eq. 1.56 for deflection corresponding to concrete cracking, steel yield and ultimate bending moments, respectively, will be as follows (Rezazadeh *et al.* 2015).

$$\delta_{cr} = \int_0^{L_s} \frac{\kappa_{cr}}{L_s} z^2 dz + \int_{L_s}^{L/2} \kappa_{cr} z dz; \quad (1.60)$$

$$\delta_y = \int_0^{L_{cr}^{(y)}} \frac{\kappa_{cr}}{L_{cr}^{(y)}} z^2 dz + \int_{L_{cr}^{(y)}}^{L_s} \left(\frac{\kappa_y - \kappa_{cr}}{L_s - L_{cr}^{(y)}} (z - L_{cr}^{(y)}) + \kappa_{cr} \right) z dz + \int_{L_s}^{L/2} \kappa_y z dz; \quad (1.61)$$

$$\delta_u = \int_0^{L_{cr}^{(u)}} \frac{\kappa_{cr}}{L_{cr}^{(u)}} z^2 dz + \int_{L_{cr}^{(u)}}^{L_y^{(u)}} \left(\frac{\kappa_y - \kappa_{cr}}{L_y^{(u)} - L_{cr}^{(u)}} (z - L_{cr}^{(u)}) + \kappa_{cr} \right) z dz + \int_{L_y^{(u)}}^{L_s} \left(\frac{\kappa_u - \kappa_y}{L_s - L_y^{(u)}} (z - L_y^{(u)}) + \kappa_y \right) z dz + \int_{L_s}^{L/2} \kappa_u z dz. \quad (1.62)$$

Fig. 1.28. Direct integration of the curvature graph: a) distinct regions of the beam (Rezazadeh *et al.* 2015); b) strain profile

The result of this solution will be similar to the graphical analytical Vereshchagin method. The basic idea of Vereshchagin method (or Vreshchagin rule) is to replace the integration procedure according to Maxwell-Mohr integral by an elementary algebraic procedure on two bending moment diagrams in the actual (M) and unit states ($\bar{M}$) (Fig. 1.29). Graph multiplication procedure (Karnovsky and Lebed, 2010):

$$\frac{1}{EI} \int M \bar{M} dx = \frac{\Omega_p y_c}{EI}. \quad (1.63)$$

The graph multiplication procedure may be presented by the conventional symbol (*) as (Karnovsky and Lebed, 2010):

$$\delta = \frac{1}{EI} \int M \bar{M} dx = \frac{M \times \bar{M}}{EI}. \quad (1.64)$$

Fig. 1.29. Graph multiplication method. Bending moment diagrams M and $\bar{M}$ in actual and unit states (Karnovsky and Lebed, 2010)

An approach based on the built-up bars theory should also be discussed. In this case, the composite flexural stiffness of concrete and FRP is used to calculate the deflection (Skuturna *et al.* 2008, Skuturna, 2009):

$$\delta = M_a \left(\frac{L^2}{8EI_{eff}} + \frac{1}{D} \left(\frac{\cosh(0.5\sqrt{\alpha\beta}L) - 1}{\alpha\beta \cosh(0.5\sqrt{\alpha\beta}L)} \right) \right), \quad (1.65)$$

where the effective flexural stiffness of concrete and FRP members:

$$EI_{eff} = E_{cm} I_{c,eff} + \frac{E_{cm} A_{c,eff} E_f A_f a^2}{E_{cm} A_{c,eff} + E_f A_f}; \quad (1.66)$$

$$\frac{1}{D} = \frac{1}{E_{cm} I_{c,eff}} - \frac{1}{EI_{eff}}. \quad (1.67)$$

The main advantage of this approach is that the composite FRP-concrete bond stiffness is evaluated. However, the effect of internal steel reinforcement is not taken into account.

1.5. Review of Techniques to Predict Cracking Response

Crack opening reduces the stiffness of the element and unfolds the way for reinforcement corrosion, therefore proper assessment of crack parameters is not less important. After concrete cracking, parts of the element separated by the crack rotate about the intersection point of the crack surface and neutral axis. Idealised representation of this phenomenon in the constant moment region with multiple cracks is given in Fig. 1.30 with mean values of crack width w_{crm} and crack spacing L_{crm} .

Fig. 1.30. Idealised mechanics based beam hinge model in a constant moment region with multiple cracks (based on Visintin *et al.* 2012)

Generally, it is accepted that the crack width will be the difference of reinforcement and concrete displacement between two adjacent cracks:

$$w_{cr} = u_R - u_c. \quad (1.68)$$

Strain expression:

$$w_{cr} = L_{cr} (\varepsilon_R - \varepsilon_{ct}). \quad (1.69)$$

Whereas the crack width limitation applies to the maximum crack width, Eurocode 2 proposes to multiply the maximum crack spacing with the difference of mean values of strain.

$$w_{EC2} = L_{cr, \max} (\varepsilon_{sm} - \varepsilon_{cm}). \quad (1.70)$$

The effect of concrete free shrinkage is added in Model Code 2010:

$$w_{MC} = L_{cr, \max} (\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}). \quad (1.71)$$

Fib bulletin 14 uses the tension stiffening effect to express the mean strain:

$$w_{fib} = 1.7 L_{crm} \varepsilon_{Rm} = 1.7 L_{crm} \varepsilon_N \left(1 - k_s k_t \left(\frac{M_{cr}}{M_a} \right)^2 \right), \quad (1.72)$$

where $k_s = 0.5$ and 1.0 for smooth and deformed steel, respectively; $k_t = 0.5$ and 1.0 for long-term and short-term loading, respectively; ε_N is the reinforcement strain in the fully cracked state. Assuming $\varepsilon_N \approx \varepsilon_{s1} \approx \varepsilon_f + \varepsilon_0$ and with $N_r = N_{s1} + N_f$, ε_N is given as (fib bulletin 14):

$$\varepsilon_N = \frac{N_r + E_f A_f \varepsilon_0}{E_{s1} A_{s1} + E_f A_f}, \quad (1.73)$$

where $N_r = M_a / z_e$ and z_e is the lever arm between the total tensile force ($N_{s1} + N_f$) and the compression force ($N_c + N_{s2}$).

The expressions of a crack spacing are provided further. Eurocode 2:

$$L_{cr, \max} = 1.7 L_{crm} = 3.4c + 0,425 k_1 k_2 \phi / \rho_{t, \text{eff}}. \quad (1.74)$$

Model Code 2010:

$$L_{cr, \max} = 2c + \frac{f_{cm} \phi}{2 \tau_{sm} \rho_{t, \text{eff}}}. \quad (1.75)$$

Fib bulletin 14, originally proposed by Rostásy *et al.* (1996):

$$L_{crm} = \frac{2 f_{ct} A_{ct, \text{eff}} \xi_b E_f A_f}{\tau_{fm} u_f (E_{s1} A_{s1} + \xi_b E_f A_f)}, \quad (1.76)$$

where the mean bond stresses of the steel and the FRP, respectively: $\tau_{sm} = 1.8 f_{ct}$ (CEB 1993) and $\tau_{fm} = 1.25 f_{ctm}$ (Holzenkämpfer, 1994); u_f and u_{s1} are the bond perimeters of FRP and steel reinforcement respectively (see Fig. 1.2); c is a thickness of concrete cover; k_1 is a coefficient which evaluates the bond properties of the bonded reinforcement: 0,8 for high bond bars and 1,6 for bars with an effectively plain surface (e.g. prestressing tendons); k_2 is a coefficient which takes into account the distribution of strain: 0,5 for bending and 1,0 for pure tension; ϕ is a diameter of steel reinforcement; $\rho_{t,eff}$ is the effective reinforcement ratio of tensile zone of concrete, in Eurocode 2 and Model Code 2010 expressed as follows.

$$\rho_{t,eff} = \frac{A_{s1} + \xi_1 A_p}{A_{ct,eff}}, \quad (1.77)$$

where A_p is the area of pre- or post-tensioned tendons within $A_{ct,eff}$; ξ_1 is the adjusted ratio of bond strength taking into account the different diameters of prestressing and reinforcing steel.

As proposed by Ceroni and Pecce (2009) for FRP strengthened RC structures, $A_p = A_f$ and $\xi_1 = E_f / E_{s1}$, then the effective reinforcement ratio of tensile zone of concrete will be as follows:

$$\rho_{t,eff} = \frac{A_{s1} + A_f E_f / E_{s1}}{A_{ct,eff}}, \quad (1.78)$$

where $A_{ct,eff}$ is the effective area of concrete in tension (Eurocode 2):

$$A_{ct,eff} = bh_{ct,eff} = \min \begin{cases} 2,5bd_1; \\ b(h - x_c)/3; \\ h/2. \end{cases} \quad (1.79)$$

Bond parameter in Eq. 1.76 (fib bulletin 14):

$$\xi_b = \frac{\tau_{fm} E_{s1} A_{s1} u_f}{\tau_{sm} E_f A_f u_{s1}}. \quad (1.80)$$

U.S. design code (ACI 224.1R-07) has relied on Frosch (1999) physical model and related crack width to the position of reinforcement:

$$w_{ACI} = 2 \frac{\sigma_{s1}}{E_{s1}} k_\beta \sqrt{d_c^2 + \left(\frac{L_{sb}}{2} \right)^2}, \quad (1.81)$$

where k_β is the ratio of the distance between the neutral and tension face to the distance between the neutral axis and centroid of reinforcing steel (approximately

$1.0 + 0.08d_c$); d_c is the thickness of cover from tension face to center of closest bar; L_{sb} is a bar spacing.

If equations 1.72 and 1.81 were aligned, then:

$$1.7L_{crm} \approx 2k_\beta \sqrt{d_c^2 + \left(\frac{L_{sb}}{2}\right)^2} \left/ \left(1 - k_s k_t \left(\frac{M_{cr}}{M_a}\right)^2\right)\right. \quad (1.82)$$

It is also proposed (Kaklauskas 2017, Kaklauskas *et al.* 2019) to divide the area between two adjacent cracks as shown in Fig. 1.31. In this case, the reinforcement debonding (l_d) and central zones (l_c) are distinguished.

Fig. 1.31. Reinforcement strain profiles: a) Kaklauskas *et al.* 2019; b) Kaklauskas 2017

Alternatively, Ramanauskas *et al.* (Ramanauskas, 2019, Ramanauskas *et al.* 2020) proposed a neural network implementation technique for estimating the primary crack spacing.

Different approach was proposed by Jokūbaitis *et al.* (Jokūbaitis and Kamaitis, 2000, Jokūbaitis, 2001, Jokūbaitis, 2009, Jokūbaitis and Juknevičius, 2009, Jokūbaitis *et al.* 2013, Jokūbaitis and Juknevičius, 2013) based fracture mechanics of solids. In accordance with his crack development model for RC structures, the flexural RC element crack has two peaks, one leads to the crack spread toward the beam neutral axis, and the other refers to the tensile reinforcement. The crack apex closer to the neutral axis opening width is critical to give rise to further crack spread, for what concrete and steel equal to the tensile strength of concrete (f_{ct}), steel reinforcement stress (σ_s) resist. Parts of the element separated by the crack rotate about the intersection point of the crack surface and neutral axis (Figure 1.32).

Fig. 1.32. The model for calculation of normal crack development (Jokūbaitis *et al.* 2013)

There was an established connection between crack width and depth based on the similarity of the triangles:

$$\frac{\delta_{cr,0}}{w_{cr}} \approx \frac{h_{ct}}{h_{cr}}, \quad (1.83)$$

where h_{cr} and h_{ct} are the crack depth and the depth of the tensile zone above it, respectively; w_{cr} and $\delta_{cr,0}$ are the crack width and the critical width of a normal crack tip respectively.

Wide range of experimental research were conducted by Jokūbaitis *et al.* (Jokūbaitis and Kamaitis, 2000, Jokūbaitis, 2001, Jokūbaitis, 2009, Jokūbaitis and Juknevičius, 2009, Jokūbaitis *et al.* 2013, Jokūbaitis and Juknevičius, 2013) and the empirical expression of the critical width of a normal crack tip was derived for flexural RC elements:

$$\delta_{cr,0} \approx 0.00012 d_1 \sqrt[3]{\phi}. \quad (1.84)$$

Furthermore, Jokūbaitis related crack parameters to reinforcing steel stress using the methods of fracture mechanics (Jokūbaitis *et al.* 2013, Jokūbaitis and Juknevičius, 2013):

$$\sigma_{s1} = 0.785 \left(\frac{M_{tot} (0.75 h_{cr} + h_{ct})}{I_{eq}} - \frac{P}{A_{eq}} \right) \left(\frac{b h_{cr}}{A_{s1}} \right)^{\frac{1}{2}} k_{\alpha} \leq f_{s1,y}; \quad (1.85)$$

$$M_{tot} = M - P(e_p - y + h_{cr} + h_{ct}), \quad (1.86)$$

where A_{eq} and I_{eq} are the area of equivalent cross-section and the second moment of it, respectively; k_a is the dimensionless adjustment function which depends on the depth of the crack and geometry of the cross-section.

Eq. 1.85 has a great potential for use in a condition assessment of existing reinforced concrete structures before strengthening.

1.6. Conclusions of Chapter 1 and Formulation of the Tasks of the Thesis

The following conclusions can be drawn based on the literature review:

1. The advantages of NSM technique over the EBR, found in the literature raise many questions. For instance, grooving is a more complicated and more time-consuming process than the surface grinding on a construction site. Moreover, the damage is made on a concrete member and there is a high risk of cutting through the transverse reinforcement. In this respect, EBR seems to be a safer and easier to install option.
2. The load-carrying capacity of the strengthened members can be calculated based on the principles of reinforced concrete design while assuming a full shear connection of concrete and FRP. The combination of the design code formulas of the FRP strain for strengthened members and prestressing tendons allows to evaluate the initial stress state and FRP prestressing effects on the strengthened beam behaviour.
3. Intermediate crack induced (IC) debonding is distinguished as the most common failure mode of FRP strengthened RC members. Therefore, methods assessing this type of failure are also applied in design codes. However, techniques found in the literature use the fracture energy/failure criterion or limit stresses derived from narrow parametric scope pull-off/out shear tests and usually work only for EBR or only for NSM strengthening techniques.
4. A proper estimation of the flexural stiffness of the strengthened member will have the greatest impact on the accuracy of the deflection prediction. Therefore, a simpler modelling of the cracked and noncracked zones of the element with springs of different stiffness is acceptable. However, evaluation of the actual flexural stiffness of the element, considering the effects of concrete-FRP partial shear connection and FRP prestressing remains an issue.
5. Usually, the crack width is expressed as the reinforcement strain multiplied by crack spacing. European design code relates the crack spacing to the concrete cover and the diameter of steel reinforcement, on the other hand, U.S. design codes use steel bar spacing instead of reinforcement

diameter. A different approach was used in the model based on the fracture mechanics of solids, where the proposed dependence of reinforcement stress and crack parameters has a great potential for use in a condition assessment of existing reinforced concrete structures before strengthening.

6. Tensioning against the strengthened member can be singled out as the most suitable FRP prestressing method for a field application. However, anchoring the FRP can cause a number of issues.

Based on the literature survey performed, the following tasks are raised:

1. To develop an analytical evaluation model of partial shear connection between concrete and FRP.
2. To develop analytical calculation models of load-carrying capacity, deflection and crack parameters of FRP strengthened RC members in bending taking into account the partial shear connection, initial stress state and prestressing effects.
3. To develop a prestressing device for externally bonded CFRP laminates, and to solve the issues of clamping and anchorage of the laminate.
4. To perform an experimental investigation on the deformation, cracking and load carrying capacity of real-scale RC beams, strengthened with prestressed CFRP laminates, subjected to a short-term load action.
5. To evaluate the adequacy of the developed analytical methods in comparison with the obtained experimental results.

Assessment of Ultimate and Serviceability States of Flexural Reinforced Concrete Members Strengthened with Prestressed Fibre Reinforced Polymer

In this chapter, a proposed analytical calculation method for the evaluation of concrete-FRP partial shear connection considering the full degradation of the bond from concrete cracking to ultimate load. Also, calculation methods regarding load carrying capacity, deflection and cracking analysis of RC beams strengthened with prestressed FRP composites. This chapter includes the material presented in journal publications by Slaitas *et al.* (2017a, 2017b, 2018a, 2018b, 2018c, 2019, 2020, 2021a, 2021b).

2.1. Partial Shear Connection

2.1.1. Intermediate Crack Induced Debonding

An analytical solution of intermediate crack induced debonding (IC) failure of RC members strengthened with FRP composites is presented. The object of consideration is the connection of two elements: concrete and FRP composites. In the case

of IC debonding, the bond slip between two adjacent cracks will be the difference of displacement (Eq. 2.1).

$$s = u_f - u_c. \quad (2.1)$$

The first derivative of the Eq. 2.1 will be the difference of strains.

$$\frac{ds}{dz} = \varepsilon_f - \varepsilon_c = \left(\frac{1}{E_f A_f} + \frac{1}{E_{cm} A_{c,eff}} + \frac{a^2}{EI} \right) T - \frac{\sum M_i}{EI} a, \quad (2.2)$$

where the effective concrete area $A_{c,eff} = bx_{c0}$ and the total flexural stiffness of two elements:

$$EI = E_{cm} I_{c,eff} + E_f I_f \approx E_{cm} I_{c,eff} = E_{cm} \frac{bx_{c0}^3}{3}, \quad (2.3)$$

where x_{c0} is the neutral axis depth of RC member before strengthening.

The material's response to shear stress can be described by the shear modulus:

$$G_{eff} = \frac{\tau_{zy}}{\gamma_{zy}} = \frac{T/A_b}{\Delta z / a} = \frac{Ta}{A_b \Delta z}, \quad (2.4)$$

where the bond area A_b will be a bond width u_f (see Fig. 1.2) multiplied by the bond length L_b .

If the bond between two adjacent cracks is analysed, the bond length will be equal to the crack spacing $L_b \approx L_{cr}$. Following the same logic, the assumption is accepted that the bond deformation Δz between two adjacent cracks will be approximately equal to the crack width w_{cr} .

Afterwards, the effective shear modulus can be written in the following form.

$$G_{eff} = \frac{Ta}{u_f L_{cr}^2 \varepsilon_N}. \quad (2.5)$$

The strain of the concrete and FRP joint, caused by the total axial force:

$$\varepsilon_N = \frac{N_r}{E_{cm} A_{ct,eff} + E_f A_f} = \frac{M_a}{y_f (E_{cm} A_{ct,eff} + E_f A_f)}, \quad (2.6)$$

where the effective concrete area in tension (based on Eurocode 2):

$$A_{ct,eff} = bh_{ct,eff} = \min \begin{cases} 2.5bd_{1,eff}; \\ b(h - x_{c0})/3; \\ bh/2. \end{cases} \quad (2.7)$$

where $d_{1,eff}$ is the depth of tensile steel and FRP reinforcement resultant (see Fig. 2.1):

$$d_{1,eff} = h - \frac{A_f \sigma_f d_f + A_{s1} \sigma_{s1} d}{A_f \sigma_f + A_{s1} \sigma_{s1}} \approx h - \frac{A_f E_f d_f + A_{s1} E_{s1} d}{A_f E_f + A_{s1} E_{s1}}. \quad (2.8)$$

Fig. 2.1. Effective tensile area of concrete: a) EBR; b) NSM

It is also assumed that a maximum transmittable force through the contact surface of concrete and FRP will be as follows.

$$T = A_f \sigma_f \approx \frac{A_f M_a y_f}{I_{cr}}, \quad (2.9)$$

where I_{cr} is the second moment of the cracked strengthened RC member's cross-section; y_f is the distance from FRP reinforcement centroid to the neutral axis of the element.

In relation to the previous section literature review, repeated corrections for stress distribution must be accepted for joint length and width in the bond analysis of concrete and FRP. The following coefficients have been adopted to take account of these adjustments (based on Model Code 2010).

$$k_L = \frac{L_b}{L_{b,eff}} \left(2 - \frac{L_b}{L_{b,eff}} \right) \leq 1; \quad (2.10)$$

$$k_b = \sqrt{\frac{2 - u_f/b}{1 + u_f/b}}; \quad (2.11)$$

$$L_{b,eff} \approx \sqrt{\frac{E_f A_f}{2 f_{ctm} u_f}}. \quad (2.12)$$

Additionally, the FRP anchorage improves the bond behaviour and its effect should be taken into account. Skuturna (2009) has tested different types of anchorage of FRP reinforcements and described its influence on concrete and FRP bond. On the basis of the researches of Skuturna (2009), Valivonis and Marciukaitis (2006), an additional factor k_a is added, taking into account the influence of anchorage of FRP reinforcement: when FRP is not anchored $k_a = 2.5$, a similar value to the one, proposed in the Model Code 2010 for intermediate sections; when FRP is anchored $k_a = 5$. If FRP reinforcement continues beyond the supports (practically impossible on the construction site, but convenient at laboratory conditions), then $k_a \rightarrow \infty$ and $\psi_f \rightarrow 1$.

The rearranged value of effective shear modulus:

$$G_{eff.B} \approx \frac{k_a k_L k_b A_f y_f^2 a (E_{cm} A_{ct,eff} + E_f A_f)}{u_f I_{cr} (\min(L_b, L_{b,eff}))^2}. \quad (2.13)$$

A further solution is processed following Section 1.3.8. The contact stiffness reduction factor for the beam loaded with two concentrated forces will be as follows.

$$\psi_{f.B} = 1 - \frac{\cosh(\sqrt{\alpha\beta}(0.5L - L_s))}{\sqrt{\alpha\beta}L_s \cosh(0.5\sqrt{\alpha\beta}L)} \sinh(\sqrt{\alpha\beta}L_s). \quad (2.14)$$

The lower limit of FRP strain is accepted equal to 50 % of the concrete-FRP full shear connection, i.e., $0.5 \leq \psi_f \leq 1.0$. Graphical illustration of the strengthened member's strain profile taking into account a concrete-FRP partial shear connection is presented in Fig. 2.2.

The bond parameter in Eq. 2.14:

$$\sqrt{\alpha\beta}_B = \sqrt{\frac{k_a k_L k_b A_f y_f^2 (E_{cm} A_{ct,eff} + E_f A_f)}{I_{cr} (\min(L_b, L_{b,eff}))^2} \left(\frac{1}{E_f A_f} + \frac{1}{E_{cm} A_{c,eff}} + \frac{a^2}{EI} \right)}. \quad (2.15)$$

The above solution not only describes the limit state of strains in the element, but also allows to evaluate concrete-FRP partial shear connection effect on deflection and crack parameters.

Fig. 2.2. Strain profile taking into account concrete-FRP partial shear connection

2.1.2. Simplified Bond Strength Solution

A simplified solution of a concrete-FRP bond strength can be derived in a similar manner as for the tensile element, not taking into account the cracking behaviour of concrete. In the case of tension or pull-off / pull-out shear tests, Eq. 2.13 gets the following simplified form.

$$G_{eff.T} \approx \frac{k_a k_L k_b a (E_{cm} A_{ct,eff} + E_f A_f)}{u_f \left(\min(L_b, L_{b,eff}) \right)^2}, \quad (2.16)$$

where L_b is the design bond length from the free end to the loaded end of FRP reinforcement.

Design bond length will be equal to the full length of a bonded FRP reinforcement for strengthened members in tension or pull-off/out shear tests $L_b = L_{bond}$, and the distance from FRP end to the mid-span of strengthened members in bending (the highest bending moment point) $L_b = L_{bond} / 2$.

Moreover, there will not be any external bending moment while adopting tensile members for derivation. In this case Eq. 1.43 takes a simpler second-order differential equation form and the bond shear stress graph adopts the exponential shape as it was presented in Fig. 1.14a [MPa·m]:

$$\tau_f(z) = F_R \sqrt{\alpha \beta} e^{-\sqrt{\alpha \beta} z}. \quad (2.17)$$

Then the bond shear force:

$$T = \int_0^{L_b} \tau_f(z) dz = F_R \left(1 - e^{-\sqrt{\alpha\beta} L_b} \right). \quad (2.18)$$

The contact stiffness reduction factor for a simplified bond strength case:

$$\psi_{f,T} = 1 - e^{-\sqrt{\alpha\beta} L_b}. \quad (2.19)$$

Bond stiffness reduction factor in Eq. 2.19 describes the amount of force transferred to the concrete member through the whole bond length.

The ultimate load of a fully rigid bond F_R will depend on the load conditions described below.

Four types of pull-off/out shear tests were analysed in this paper: single lap shear test SLST (Fig. 2.3a), bending shear test BST (Fig. 2.3b), double lap shear test DLST (Fig. 2.3c) and push-pull shear test PPST (Fig. 2.3d).

Fig. 2.3. Pull-off/out shear tests: a) single lap shear SLST; b) bending shear BST; c) double lap shear DLST; d) push-pull shear PPST

The equilibrium condition of forces expressed through FRP strain in the first 3 cases (Fig. 2.3a, b, c):

$$\eta \lambda f_{cm} b d_f \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_f} = A_f E_f \varepsilon_f. \quad (2.20)$$

FRP strain from Eq. 2.20:

$$\varepsilon_f = \frac{\sqrt{(A_f E_f \varepsilon_{cu})^2 + 4 A_f E_f \eta \lambda f_{cm} b d_f \varepsilon_{cu}} - A_f E_f \varepsilon_{cu}}{2 A_f E_f} \leq \frac{f_f}{E_f}. \quad (2.21)$$

Then the depth of neutral axis:

$$x_c = \frac{A_f E_f \varepsilon_f}{\eta \lambda f_{cm} b}. \quad (2.22)$$

Afterwards, the ultimate transferable force through the bond:

$$F_u = A_f E_f \varepsilon_f \psi_{f,T}. \quad (2.22)$$

Limitation for EBR systems:

$$F_u = A_f E_f \varepsilon_f \psi_{f,T} \leq k_L k_b b_f L_{b,eff} f_{cm}. \quad (2.23)$$

In the case of the push-pull shear test (see Fig. 2.3d), the depth x_c is equal to the pushing block height and FRP strain will be as follows:

$$\varepsilon_f = \frac{\eta \lambda f_{cm} b x_c}{A_f E_f} \leq \frac{f_f}{E_f}. \quad (2.24)$$

The ultimate transferable force through the bond is calculated by Eqs. 2.22, 2.23

Different test data covering the test methods discussed above were taken for the pull-off/out shear test verification (see Table 2.1). Four different types of shear tests were analysed: single lap shear, bending shear, double lap shear, and push-pull shear tests. The concrete blocks were strengthened with externally bonded and near surface mounted carbon fibre, glass fibre, basalt fibre sheets, laminates, strips and rods. The comparison of the numerical and experimental ultimate pull-off/out shear forces (calculated according to Eqs. 2.22, 2.23) is presented in Figure 2.4.

A great variety of different pull-off/out shear test combinations were analysed and with low systematic error (1.18), low random error (0.25), low coefficient of variation (21.32 %) and high coefficient of correlation (0.91) it can be concluded that the ultimate pull-off/out shear load was predicted sufficiently precisely. Of course, there are some points with higher errors, which slightly worsens the statistics, but those errors are on the safety side. It means that some elements had better bond conditions than the predicted one. In fact, in some cases the same samples can show an absolutely different result, therefore, the calculation should be directed to the safety side.

Table 2.1. Main parameters of pull-off/out shear tests

Reference	A_f/bd_f , %	L_b , mm	f_{cm} , MPa	f_f , MPa	E_f , GPa	EBR/NSM	Test type
Bilota <i>et al.</i> 2011	0.05–0.53	300	19	1250–3194	46–221	EBR/NSM	SLST
Sena Cruz and Barros, 2004	0.08	40–80	33–70	2740	158	NSM	BST
Torres <i>et al.</i> 2016	0.17	48–240	23	1350–2350	64–170	NSM	SLST
Novidis <i>et al.</i> 2007	1.24–1.27	60–120	35	2108	124	NSM	SLST
Costa and Barros, 2011	0.06–0.12	40–300	25–41	2833–3023	156–171	NSM	PPST, DLST, BST
Diab and Farghal, 2014	0.17–0.5	250	40	2100–3400	91–230	EBR	DLST
Yao <i>et al.</i> 2005	0.01–0.43	75–240	19–27	351–4114	23–256	EBR	PPST
Bilota <i>et al.</i> 2016	0.06–0.39	300	30–42	1208–3536	48–187	NSM	DLST

Mean values were taken for the same samples.

Fig. 2.4. Experimental-calculated pull-off/out ultimate load

2.2. Load-Carrying Capacity

As it was discussed in a first chapter, high plastic deformations occur in the element after the tensile steel reinforcement reaches the yield stress. Therefore, the compressive stress diagram of concrete should be treated as nonlinear, which can

$$B_{u2} = A_{s2}f_{s2,y} + A_f\psi_f E_f (\varepsilon_{cu} + \varepsilon_0 - \varepsilon_p) - A_{s1}f_{s1,y}; \quad (2.31)$$

$$C_{u2} = A_f\psi_f E_f \varepsilon_{cu} d_f. \quad (2.32)$$

Third, steel yielding followed by FRP fracture, while compression reinforcement does not reach the yield stress, $\sigma_{s2} \leq f_{s2,y}$ and $\sigma_f > f_f$.

$$B_{u3} = A_{s2}E_{s2}\varepsilon_{cu} - A_{s1}f_{s1,y} - A_f\psi_f f_f; \quad (2.33)$$

$$C_{u3} = A_{s2}E_{s2}\varepsilon_{cu} d_2. \quad (2.34)$$

Fourth, steel yielding followed by FRP fracture, while compression reinforcement reaches the yield strength, $\sigma_{s2} > f_{s2,y}$ and $\sigma_f > f_f$.

$$x_{c,u} = \frac{A_{s1}f_{s1,y} + A_f\psi_f f_f - A_{s2}f_{s2,y}}{\eta f_{cm} b \lambda}. \quad (2.35)$$

Reduction factors of concrete compressive stress diagram η , λ can be found by applying Dulinkas *et al.* (2009) methodology (section 1.2.) with the following values of ultimate concrete strain and the area of nonlinear concrete compressive stress diagram, respectively:

$$\varepsilon_{c2} = \left(\frac{\sigma_f}{E_f} + \varepsilon_0 - \varepsilon_p \right) \frac{x_{cu}}{d_f - x_{cu}} \leq \varepsilon_{cu}; \quad (2.36)$$

$$A_{sum} = \frac{A_{s1}f_{s1,y} + A_f\psi_f \sigma_f - A_{s2}\sigma_{s2}}{b}. \quad (2.37)$$

The use of other techniques described in Section 1.2. is also possible.

Finally, the load-carrying capacity of the strengthened member takes the following expression.

$$M_u = A_{s1}f_{s1,y} \left(d - \frac{\lambda x_{c,u}}{2} \right) + A_{s2}\sigma_{s2} \left(\frac{\lambda x_{c,u}}{2} - d_2 \right) + A_f\psi_f \sigma_f \left(d_f - \frac{\lambda x_{c,u}}{2} \right). \quad (2.38)$$

The calculation flow graph in Fig. 2.6 represents the techniques for load-carrying capacity of concrete elements strengthened with FRP subjected to bending or tension.

Fig. 2.6. Calculation flow graph of the load-carrying capacity of strengthened members

Two sudden and brittle FRP debonding failure modes of the strengthened elements are being considered: intermediate crack induced debonding or IC debonding ($\psi_{f,B}$), and simplified bond strength failure ($\psi_{f,T}$). The second technique derived for concrete elements strengthened with FRP subjected to tension and can be used for a prediction of the load carrying capacity of strengthened members in bending as well.

2.3. Deflection

2.3.1. General Assumptions

The proposed model is based on the following assumptions: 1) the cross-section of RC member remains homogenous and the dimensions do not change after loading, the infinitesimal strain theory is valid; 2) cross-section of the element remains

plane and perpendicular to the element axis during the process of deformation of RC member, yet the hypothesis of plane cross-sections is invalid for FRP reinforcement after concrete cracking point; 3) perfect bond exists between internal steel reinforcement and concrete; 4) linear-elastic properties are assumed for FRP and for steel up to yield stress, afterwards steel reinforcement is assumed as perfectly plastic material.

Generally, the moment-deflection or moment-curvature dependency graph can be separated into 5 stages (see Fig. 2.7): 1) pre-cracking $M_a \leq M_{cr}$; 2) from crack opening to strengthening moment $M_{cr} < M_a \leq M_0$; 3) strengthening/pre-stressing of FRP reinforcement $M_a = M_0$; 4) from strengthening to steel yield moment $M_0 < M_a \leq M_y$; 5) from steel yield to ultimate moment $M_y < M_a \leq M_u$.

Fig. 2.7. Moment-curvature dependency graph

Deflection of the element is a multilinear process, i.e., a new stage of development begins at the end of the previous one. However, the deflection formed in a previous stage does not change according to the new law of development, only the deflection increment can be evaluated according to the new law. Thus, in this chapter the deflection response is described in all stages presented in Fig. 2.7. Whereas the variable in general expression of deflection in Eq. 1.59 is curvature, the focus will be on it. Deflection parameters in the initial stage are estimated by general linear elastic analysis, as it is proposed in design codes (Eurocode 2, Model Code 2010, ACI 318-14) with one exception. When the element is already cracked before loading, it is assumed that the cracked and uncracked zones will be evenly distributed from the beginning of loading, i.e. $(M_{cr} / M_a)^m \leq 0.5$.

2.3.2. Post-Cracking Stage

A triangular concrete stress diagram is used for analysis at this stage. (see Fig. 2.8).

Fig. 2.8. Stress state in a cracked cross-section, post-cracking stage: a) EBR; b) NSM

The depth of neutral axis is calculated from equilibrium condition of forces and strain compatibility:

$$\frac{1}{2}bx_c^2 + \alpha_{s2}A_{s2}(x_c - d_2) + \psi_f\alpha_fA_f\left(\frac{\varepsilon_0 - \varepsilon_p}{\varepsilon_c}\right)x_c = \alpha_{s1}A_{s1}(d - x_c) + \psi_f\alpha_fA_f(d_f - x_c). \quad (2.39)$$

If the following expressions of strain and second moments of cracked concrete section area are accepted, Eq. 2.39 can be simplified.

$$\varepsilon_c \approx \frac{M_a x_c}{E_{cm} I_c}; \quad \varepsilon_0 \approx \frac{M_0 (d_f - x_0)}{E_{cm} I_{c0}}; \quad \varepsilon_p \approx \frac{M_p (d_f - x_0)}{E_{cm} I_{c0}}; \quad I_c \approx I_{c0}; \quad (2.40)$$

$$\frac{\varepsilon_0 - \varepsilon_p}{\varepsilon_c} \approx \frac{(M_0 - M_p)(d_f - x_0)}{M_a x_c}. \quad (2.41)$$

Strain at the most compressed concrete fibre of the element:

$$\varepsilon_c = \frac{M_a - M_p \psi_f + A_f E_f \psi_f \varepsilon_0 \left(d_f - \frac{x_c}{3}\right)}{\left[A_{s1} E_{s1} \left(\frac{d - x_c}{x_c}\right) \left(d - \frac{x_c}{3}\right) + A_{s2} E_{s2} \left(\frac{x_c - d_2}{x_c}\right) \left(\frac{x_c}{3} - d_2\right) + A_f E_f \psi_f \left(\frac{d_f - x_c}{x_c}\right) \left(d_f - \frac{x_c}{3}\right) \right]}. \quad (2.42)$$

General flexural stiffness of the element at this stage taking into account the tension stiffening effect (corresponding to springs in series):

$$EI_y = \frac{EI_{cr,y}}{1 - \left(\frac{M_{cr}}{M_a} \right)^m \left(1 - \frac{EI_{cr,y}}{E_{cm} I_g} \right)}. \quad (2.43)$$

Flexural stiffness of the cracked cross-section (corresponding to springs in parallel):

$$EI_{cr,y} = E_{cm} (\epsilon_c) \frac{bx_c^3}{3} + E_{s1} A_{s1} (d - x_c)^2 + (E_{s2} - E_{cm} (\epsilon_c)) A_{s2} (x_c - d_2)^2 + \psi_f E_f A_f (d_f - x_c)^2. \quad (2.44)$$

Deformation modulus of concrete is not a constant and it depends on the stress state of the element. Secant modulus E_{cm} is valid up to a concrete stress equal to 40 % of the compressive strength, afterwards its value will decrease. Concrete strain-deformation modulus relation graph based on the design code (Eurocode 2, Model Code 2010) nonlinear analysis is presented in Fig. 2.9.

Fig. 2.9. Concrete strain-deformation modulus relation

Deformation modulus of the most compressed concrete layer of the element:

$$E_c (\epsilon_c) = \frac{\sigma_c}{\epsilon_c} = f_{cm} \left(\frac{k \epsilon_{c1} - \epsilon_c}{\epsilon_{c1}^2 + (k - 2) \epsilon_{c1} \epsilon_c} \right). \quad (2.45)$$

However, Eq. 2.45 represents a peak value and to evaluate the flexural stiffness of the whole compressive zone – mean value is required (Eq. 2.46).

$$E_{cm}(\varepsilon_c) = \frac{E_c(\varepsilon_c) + E_c(0)}{2} = \frac{f_{cm}}{2} \left(\frac{k\varepsilon_{c1} - \varepsilon_c}{\varepsilon_{c1}^2 + (k-2)\varepsilon_{c1}\varepsilon_c} + \frac{k}{\varepsilon_{c1}} \right). \quad (2.46)$$

Distribution of concrete deformation modulus in the compressive zone is presented in Fig. 2.10.

Fig. 2.10. Representation of deformation modulus in a compression zone of concrete:
a) strain profile; b) deformation modulus

Curvature of the element:

$$\kappa_y = \kappa_0 - \Delta\kappa_p + \Delta\kappa_y = \frac{M_0 - M_p \psi_f}{EI_0} + \frac{M_a - M_0}{EI_y}. \quad (2.47)$$

If $M_p > M_0$, the flexural stiffness of the element will not be higher than EI_p until the moment of decompression is reached. Following this, the expression of curvature would be as follows.

$$\kappa_y = \begin{cases} \frac{M_a - M_p}{EI_p}, & M_a \leq M_p; \\ \frac{M_a - M_p \psi_f}{EI_y}, & M_a > M_p; \end{cases} \quad (2.48)$$

$$EI_p = \begin{cases} E_{cm} I_g, & M_p \leq M_{cr} - M_p; \\ \frac{EI_{cr.p}}{1 - \left(\frac{M_{cr} - M_p}{M_p} \right)^m \left(1 - \frac{EI_{cr.p}}{E_{cm} I_g} \right)}, & M_p > M_{cr} - M_p, \end{cases} \quad (2.49)$$

where flexural stiffness of the cracked cross-section:

$$EI_{cr,p} = E_{cm} \frac{bx_c^3}{3} + (E_{s1} - E_{cm}) A_{s1} (x_c - d_1)^2 + E_{s2} A_{s2} (h - d_2 - x_c)^2. \quad (2.50)$$

2.3.3. Post-Yielding Stage

First of all, the yield moment of steel reinforcement is required. It can be found using the same calculation scheme as for load-carrying capacity in Fig. 2.5. Equilibrium condition of forces and strain compatibility at the yield point of steel reinforcement:

$$\eta \lambda b x_{c,y}^2 + \alpha_{s2} A_{s2} (x_{c,y} - d_2) + \psi_f \alpha_f A_f \left(\frac{\varepsilon_0 - \varepsilon_p}{\varepsilon_{s1,y}} \right) (d - x_{c,y}) = \alpha_{s1} A_{s1} (d - x_{c,y}) + \psi_f \alpha_f A_f (d_f - x_{c,y}). \quad (2.51)$$

Stresses of compressive steel and FRP reinforcement:

$$\sigma_{s2} = E_{s2} \frac{f_{s1,y}}{E_{s1}} \left(\frac{x_{c,y} - d_2}{d - x_{c,y}} \right) \leq f_{s2,y}; \quad (2.52)$$

$$\sigma_f = E_f \left(\frac{f_{s1,y}}{E_{s1}} \left(\frac{d_f - x_{c,y}}{d - x_{c,y}} \right) - \varepsilon_0 + \varepsilon_p \right) \leq f_f. \quad (2.53)$$

Yield moment of steel reinforcement:

$$M_y = A_{s1} f_{s1,y} \left(d - \frac{\lambda x_{c,y}}{2} \right) + A_{s2} \sigma_{s2} \left(\frac{\lambda x_{c,y}}{2} - d_2 \right) + A_f \psi_f \sigma_f \left(d_f - \frac{\lambda x_{c,y}}{2} \right). \quad (2.54)$$

The depth of the neutral axis at this stage is found from the following equilibrium condition of forces and strain compatibility:

$$\eta \lambda b x_c^2 + \alpha_{s2} A_{s2} (x_c - d_2) + \psi_f \alpha_f A_f \frac{M_0 - M_p}{M_a} (d_f - x_0) = \alpha_{s1} A_{s1} \frac{M_y}{M_a} (d - x_c) + \psi_f \alpha_f A_f (d_f - x_c). \quad (2.55)$$

Concrete strain:

$$\varepsilon_c = \varepsilon_{c,y} + \frac{M_a - M_y}{A_{s2}E_{s2} \left(\frac{x_c - d_2}{x_c} \right) \left(\frac{\lambda x_c}{2} - d_2 \right) + A_f E_f \Psi_f \left(\frac{d_f - x_c}{x_c} \right) \left(d_f - \frac{\lambda x_c}{2} \right)}. \quad (2.56)$$

Flexural stiffness of the element at this stage:

$$EI_u = \frac{EI_{cr,u}}{1 - \left(\frac{M_{cr}}{M_a} \right)^m \left(1 - \frac{EI_{cr,u}}{E_{cm} I_g} \right)}. \quad (2.57)$$

Flexural stiffness of the cracked cross-section:

$$EI_{cr,u} = E_{cm}(\varepsilon_c) \frac{bx_c^3}{3} + (E_{s2} - E_{cm}(\varepsilon_c)) A_{s2} (x_c - d_2)^2 + \Psi_f E_f A_f (d_f - x_c)^2. \quad (2.58)$$

Curvature of the element:

$$\kappa_u = \kappa_0 - \Delta\kappa_p + \Delta\kappa_y + \Delta\kappa_u = \frac{M_0 - M_p \Psi_f}{EI_0} + \frac{M_y - M_0}{EI_y} + \frac{M_a - M_y}{EI_u}. \quad (2.59)$$

If $M_p > M_0$, then:

$$\kappa_u = \frac{M_a - M_p \Psi_f}{EI_y} + \frac{M_a - M_y}{EI_u}. \quad (2.60)$$

Usually, the full shear connection of concrete and FRP is taken into account for a prediction of deflection, the opposite is done in the current paper. The hypothesis of plane cross-sections is valid only until the concrete cracking point for FRP reinforcement, afterwards, FRP stiffness and transferred strain are reduced with the factor derived in Eq. 2.14.

2.4. Crack Parameters

2.4.1. Crack Model

Original fracture mechanics of solids based crack model (Jokūbaitis *et al.* 2013, Jokūbaitis and Juknevičius, 2013), discussed in chapter 1.5, was proposed for RC

elements. Yet the effect of FRP reinforcement can be added as it is proposed in Fig. 2.11a and such a model can be aligned with the strain profile in Fig. 2.11b.

Fig. 2.11. The relation of crack parameters and strains: a) crack model; b) strain profile

There are two triangles in Fig. 2.11a, one represents the tensile zone of concrete with depth h_{ct} and width δ_{cr} and the second one - idealized crack with depth h_{cr} and width w_{cr} . Also, there are two triangles in the strain profile (Fig. 2.11b). The first one refers to the concrete cracking strain ($\epsilon_{ct,max}$) and the second one refers to reinforcement strains ϵ_{sl} or ϵ_f . The flexural crack and tensile zone of concrete depths directly relate the crack parameters to strains in the element. This relation is expressed by the similarity of triangles:

$$\frac{h_{ct}}{h_{cr} - d_1} = \frac{\delta_{cr}}{w_{cr} - \delta_{cr}}; \quad (2.61)$$

$$\frac{h_{ct}}{h_{cr} - d_1} = \frac{\epsilon_{ct,max}}{\epsilon_{sl} - \epsilon_{ct,max}}. \quad (2.62)$$

Crack width from equations 2.61, 2.62:

$$w_{cr} = \frac{\delta_{cr}(\epsilon_{sl} - \epsilon_{ct,max})}{\epsilon_{ct,max}} + \delta_{cr} = \frac{\delta_{cr}\Delta\epsilon_{sl}}{\epsilon_{ct,max}} + \delta_{cr}. \quad (2.63)$$

Modified expression of a crack tip referring to beam neutral axis width for strengthened structures:

$$\delta_{cr} \approx 0.00012 d_{1,eff} \sqrt[3]{\phi_{eq}}, \quad (2.64)$$

where $d_{1,eff}$ and ϕ_{eq} are the depth of tensile steel and FRP reinforcement resultant (in mm, Eq. 2.8) and the equivalent factor of tensile zone, respectively (in mm), which can be derived by equating the mean crack spacing value proposed in Eurocode 2 (for not strengthened elements, Eq. 1.75) to the one in fib bulletin 14 (for

strengthened elements, Eq. 1.77). For strengthened flexural concrete elements reinforced with deformed steel bars:

$$\phi_{eq} = \frac{8\rho_{t,eff}A_{ct,eff}f_{ct}\xi_b E_f A_f}{\tau_{fm}u_f(E_{s1}A_{s1} + \xi_b E_f A_f)k_1k_2} = \frac{20(E_{s1}A_{s1} + E_f A_f)}{E_{s1}(1.8u_{s1} + 1.25u_f)}. \quad (2.65)$$

If the similarity of the triangles is applied at the level of FRP and compared with the general expression of the crack width, the following equation would be obtained.

$$L_{cr}\epsilon_f \approx \frac{\epsilon_f \delta_{cr}}{\epsilon_{ct,max}} \Rightarrow L_{cr} \approx \frac{\delta_{cr}}{\epsilon_{ct,max}} \approx \frac{E_{cm}\delta_{cr}}{f_{ctm}}. \quad (2.66)$$

Eq. 2.66 represents the crack spacing corresponding to the proposed crack development model.

2.4.2. Stages of Crack Width Development

Crack width can be related to the deflection of the beam through strain, hence the stages of crack width development will be similar to deflection (see Fig. 2.12): 1) from crack opening to strengthening moment $M_{cr} < M_a \leq M_0$; 2) strengthening/pre-stressing of FRP reinforcement $M_a = M_0$; 3) from strengthening to steel yield moment $M_0 < M_a \leq M_y$; 4) from steel yield to ultimate moment $M_y < M_a \leq M_u$.

Fig. 2.12. Bending moment-crack width dependency graph

General linear elastic analysis is used at the initial stage ($M_{cr} < M_a \leq M_0$). Strain increment in steel reinforcement after concrete cracking:

$$\Delta\epsilon_{s1,0} = \frac{M_a - M_{cr}}{A_{s1}E_{s1}\left(d - \frac{x_0}{3}\right) + A_{s2}E_{s2}\left(\frac{x_0 - d_2}{d - x_0}\right)\left(\frac{x_0}{3} - d_2\right)}. \quad (2.67)$$

Then initial crack width:

$$w_{cr.0} = \frac{\delta_{cr.0} \Delta \varepsilon_{s1.0}}{\varepsilon_{ct.max}} + \delta_{cr.0}. \quad (2.68)$$

Initial crack width should be checked in situ before strengthening. Axial pre-stressing force will try to close the initial crack, but the tensile steel reinforcement will resist to it. Consequently, the strain reduction due to prestress:

$$\Delta \varepsilon_p = \frac{\psi_f P}{A_{s1} E_{s1}}. \quad (2.69)$$

Afterwards, the crack width reduction will be as follows.

$$\Delta w_{cr.p} = \frac{\delta_{cr} \Delta \varepsilon_p}{\varepsilon_{ct.max}}. \quad (2.70)$$

A further increase in deformation due to load increment will be found by the following equation.

$$\Delta \varepsilon_{s1.y} = \frac{M_a - M_0}{\left[A_{s1} E_{s1} \left(d - \frac{x_c}{3} \right) + A_{s2} E_{s2} \left(\frac{x_c - d_2}{d - x_c} \right) \left(\frac{x_c}{3} - d_2 \right) + A_f \psi_f E_f \left(\frac{d_f - x_c}{d - x_c} \right) \left(d_f - \frac{x_c}{3} \right) \right]}. \quad (2.71)$$

Crack width increment:

$$\Delta w_{cr.y} = \frac{\delta_{cr} \Delta \varepsilon_{s1.y}}{\varepsilon_{ct.max}}. \quad (2.72)$$

The last stage is post-yielding, the strain and crack width increment at this stage can be expressed by the following equations.

$$\Delta \varepsilon_{s1.u} = \frac{M_a - M_y}{A_{s2} E_{s2} \left(\frac{x_c - d_2}{d - x_c} \right) \left(\frac{\lambda x_c}{2} - d_2 \right) + A_f E_f \left(\frac{d_f - x_c}{d - x_c} \right) \left(d_f - \frac{\lambda x_c}{2} \right)}; \quad (2.73)$$

$$\Delta w_{cr.u} = \frac{\delta_{cr} \Delta \varepsilon_{s1.u}}{\varepsilon_{ct.max}}. \quad (2.74)$$

Total crack width:

$$w_{cr} = w_{cr.0} - \Delta w_{cr.p} + \Delta w_{cr.y} + \Delta w_{cr.u}. \quad (2.75)$$

If the element was strengthened before loading ($M_0 = 0$), Eq. 2.75 can be simplified in the following way.

$$w_{cr} = \frac{\delta_{cr} (\Delta \varepsilon'_{s1.y} + \Delta \varepsilon_{s1.u})}{\varepsilon_{ct.max}}; \quad (2.76)$$

$$\Delta \varepsilon'_{s1.y} = \frac{M_a - M_{cr} - 2M_p}{\begin{bmatrix} A_{s1}E_{s1} \left(d - \frac{x_c}{3} \right) + A_{s2}E_{s2} \left(\frac{x_c - d_2}{d - x_c} \right) \left(\frac{x_c}{3} - d_2 \right) + \\ A_f \Psi_f E_f \left(\frac{d_f - x_c}{d - x_c} \right) \left(d_f - \frac{x_c}{3} \right) \end{bmatrix}}. \quad (2.77)$$

Relation of crack parameters to the reinforcement strains presented above allows to evaluate stress-strain state in the element from the measured maximum crack width in the normal section by solving the reverse task. Strain increment after concrete cracking:

$$\Delta \varepsilon_{s1} \approx \frac{w_{cr} - \delta_{cr}}{\delta_{cr}} \varepsilon_{ct.max}. \quad (2.78)$$

Total strain of steel reinforcement is the sum of the strain corresponding to the cracking moment with the strain increment in Eq. 2.78.

$$\varepsilon_{s1} \approx \varepsilon_{ct.max} + \Delta \varepsilon_{s1} = \varepsilon_{ct.max} \left(1 + \frac{w_{cr} - \delta_{cr}}{\delta_{cr}} \right). \quad (2.79)$$

Stress in steel and FRP reinforcements:

$$\sigma_{s1} = E_{s1} \varepsilon_{ct.max} \left(1 + \frac{w_{cr} - \delta_{cr}}{\delta_{cr}} \right) \leq f_{s1.y}; \quad (2.80)$$

$$\sigma_{s2} = E_{s2} \varepsilon_{ct.max} \left(1 + \frac{w_{cr} - \delta_{cr}}{\delta_{cr}} \right) \left(\frac{x_c - d_2}{d - x_c} \right) \leq f_{s2.y}; \quad (2.81)$$

$$\sigma_f = E_f \left(\varepsilon_{ct.max} \left(1 + \frac{w_{cr} - \delta_{cr}}{\delta_{cr}} \right) \left(\frac{d_f - x_c}{d - x_c} \right) - \varepsilon_0 + \varepsilon_p \right) \leq f_f. \quad (2.82)$$

Equations 2.78–2.82 directly relate the crack width to the element's stress-strain state.

2.5. Conclusions of Chapter 2

Within this Chapter, the following conclusions can be drawn:

1. Partial shear connection of concrete and FRP reinforcement can be taken into account by reducing the axial stiffness of FRP reinforcement with the reduction factor based on built-up bars theory. The application of this theory is greatly influenced by the finding of the effective shear modulus of the joint.
2. Bond length of FRP reinforcement is usually taken as a single uncracked zone length in force transfer research. Therefore, while analysing the bond behaviour between two adjacent cracks, the bond length could be accepted as a crack spacing, and the slip in a contact surface of concrete and FRP reinforcement could be assumed approximately equal to the concrete crack width.
3. The effective shear modulus of the joint was derived from the general expression of the shear modulus in material science with the following assumptions: the bond length is equal to the crack spacing, the slip is equal to the crack width, the FRP reinforcement and the effective tensile concrete zone will resist to the axial force acting on the joint, the maximum possible shear force is equal to the force in the FRP reinforcement.
4. Alternatively to the design codes, new methods for calculating load-carrying capacity, deflection and crack parameters of RC members strengthened with prestressed FRP composites using a novel approach to concrete-FRP partial shear connection effect have been proposed.
5. Crack width can be related to the deflection of the beam through strain, hence the stages of crack width development will be similar to deflection: 1) from crack opening to the strengthening moment $M_{cr} < M_a \leq M_0$; 2) strengthening/prestressing of FRP reinforcement $M_a = M_0$; 3) from the strengthening to steel yield moment $M_0 < M_a \leq M_y$; 4) from steel yield to ultimate moment $M_y < M_a \leq M_u$.
6. The direct relation of concrete crack parameters and strains is proposed based on the modified fracture mechanics crack model. The mathematically and physically correct application of the proposed relation is presented in two possible ways: prediction of the maximum crack width development and stress-strain state evaluation – reverse task.

Experimental Tests and Validation of Theoretical Load-Carrying Capacity, Deflection and Cracking Response Models of the Strengthened Reinforced Concrete Members

This Chapter presents an experimental investigation of FRP prestressing and concrete-FRP partial shear connection effects on load carrying capacity, cracking, and short-term deformation of RC beams strengthened with CFRP laminates. Experimental results on tests of 14 RC beams (all casted at once) having constant reinforcement ratio, but different levels of prestressing are reported. Moreover, a prestressing device of CFRP laminates developed by the author is presented (European patent application No. EP21167213.4). A comparative analysis of experimental and analytical load-carrying capacity, deflection and crack widths is also presented. In addition to the author's research, the results used in the analysis were collected from different scientific publications. This chapter includes the material presented in journal publications by Slaitas *et al.* (2018a, 2018b, 2018c, 2019, 2020, 2021a, 2021b).

Beam numbers were assigned according to the following logic XX-YY-Z, where XX indicates the beam series: CB – control beam, BS – beam strengthened before external load action, and BSM – beam strengthened under external load action; YY – shows the level of prestressing in percent of load bearing capacity of FRP reinforcement; Z used in the case of twin beams. Beams strengthened not less than 174 days after concrete casting and tested 7 days after strengthening. The crack width measured at the level of the reinforcement in the pure bending zone with the digital concrete crack width gauge BJQF-1 (measurement range 0.01 – 6.5 mm, accuracy $\leq \pm 0.02$ mm) presented in Fig. 3.3. Crack width measured with the step of $\approx 0.1 M_u$ after the concrete cracking point.

Fig. 3.3. Digital concrete crack width gauge BJQF-1

Deflection of the beam was measured at 5 control points (mid-span, loading points, supports) with 100 mm base LVDT's (I1-I5), strains (L1-L15) and anchorage zone slip (L16-L19) with 25 mm base LVDT's. Principle layout of the measurement devices is given in Fig. 3.4. Before strengthening, all beams were preloaded with the load equal to 40–50 % of load-carrying capacity of the element and unloaded. Afterwards, LVDT's mounted at potentially critical cracks.

Fig. 3.4. Principle layout of the measurement devices

Concrete density of 36 cubes was 2245.07 ± 33.47 kg/m³. Water / cement ratio of the beams was 0.49, the particle size distribution of concrete aggregates is presented in Table 3.1.

Table 3.1. Particle size distribution of concrete aggregates

Aggregate	Proportion, %	Fraction by particle size mm, %								
		<0.125	0.125–0.25	0.25–0.5	0.5–1	1–2	2–4	4–8	8–16	16–31.5
Sand	47	1.2	1.8	11.7	13.7	13.1	5	0.5	0	0
Gravel	53	0.2	0	0.1	0	0.6	6.1	24.2	21.3	0.5

The properties of the materials were determined in parallel with the beam tests. Since the first beam test was 174 days after concrete casting, no change in material properties over the testing time was detected. Therefore, the averages of the full sample are presented in Table 3.2.

Compressive strength of concrete was found from the compressive tests of 100×100×100 mm concrete cubes (Fig. 3.5a) and 150 mm diameter, 300 mm height concrete cylinders (Fig. 3.5b) (according to ISO 1920-4 requirements). Secant modulus of elasticity was found from the tests of the same cylinders, using method B from LST EN 12390-13:2014.

Fig. 3.5. Compressive tests of concrete: a) cubes; b) cylinders

Tensile strength of concrete was found from the flexural tests of 100×100×400 mm concrete prisms (Fig. 3.6a) and the same prism splitting tests (Fig. 3.6b) (according to ISO 1920-4 requirements).

Fig. 3.6. Determination of tensile strength of concrete: a) flexural test; b) concrete splitting test

Table 3.2. Concrete specimen test results

Test type	Specimen	Mean strength/secant modulus, MPa	Standard deviation, MPa	Coefficient of variation, %	Number of samples
Comprre-sion	Cube, $f_{cm.cube}$	50.81	5.00	9.85	36
Comprre-sion	Cylinder, f_{cm}	50.00	4.22	8.43	17
Comprre-sion	Cylinder, E_{cm}	35409	4494	12.69	17
Flexural	Prism, $f_{ctm.fl}$	8.16	0.62	7.59	26
Concrete splitting	Prism, $f_{ctm.sp}$	7.11	0.88	12.39	52

Mean axial tensile strength of concrete is found from flexural tests by the following equation (Model Code 2010):

$$f_{ctm} = f_{ctm.fl} \frac{0.06h^{0.7}}{1 + 0.06h^{0.7}}. \quad (3.1)$$

From concrete splitting test (Model Code 2010):

$$f_{ctm} = \alpha_{sp} f_{ctm.sp}, \quad (3.2)$$

where the conversion factor α_{sp} varies from 0.67 to 0.95, the recommended value in Model Code 2010 is 1.00. If the values from equations 3.1 and 3.2 were aligned, then $\alpha_{sp} = 0.69$.

The value of tensile strength of concrete obtained from Eq. 3.1 corresponds to the U.S. design code tensile strength value ($f_{ctm} \approx 0.623\sqrt{f_{cm}}$) for normal strength concrete (according to ACI 318-14) with 11 % reserve.

Strengthening material was S&P C-Laminate bonded to the concrete surface with S&P Resin 220. Tensile tests of the laminate (in accordance with ACI 440.3R-33), adhesive and steel reinforcement are presented in Fig. 3.7, and the results in Table 3.3. Initial bending moments and FRP prestressing stress of the tested beams are presented in Table 3.4.

Fig. 3.7. Tensile tests: a) CFRP laminate; b) epoxy resin; c) steel reinforcement

Table 3.3. Reinforcement properties

Parameter	CFRP laminate A_f , 1.2 x 50 mm	Tensile steel reinforcement A_{s1} , 2Ø14	Compressive steel reinforcement A_{s2} , 2Ø10	Epoxy resin
Tensile strength / yield strength for steel f_i , MPa	2627.61	568.50	537.68	9.20
Modulus of elasticity E_i , GPa	170.06	199.75	195.55	10.49

Table 3.4. Prestressing stress and initial bending moments of the tested beams

Beam ID	CFRP prestressing stress σ_p , MPa	Initial bending moment M_0 , kNm
BS-0	0	0
BS-20	522.08	0
BS-22	568.57	0
BS-48	1259.01	0
BS-49	1287.07	0
BSM-0-1	0	30.67
BSM-0-2	0	31.03
BSM-17	447.82	32.86
BSM-18	482.12	29.74
BSM-35	919.17	29.16
BSM-46	1212.24	29.97
BSM-56	1471.59	29.72

3.1.2. Prestressing System

A prestressing system was developed by the author and used in experimental research (European patent application No. EP21167213.4). The proposed FRP prestressing system is based on tensioning against the strengthened beam and includes three main items: a clamping unit, a light frame for the hydraulic jack, and anchoring plates (see Fig. 3.8).

Fig. 3.8. Prestressing system of CFRP laminate: a) schematic side view; b) schematic bottom view; c) picture of the prototype

Whereas the steel frame for the hydraulic jack is tightened with slender bolts, an additional steel frame anchor system has been adopted to achieve higher tension levels (see Fig. 3.9).

Fig. 3.9. Additional anchor system: a) schematic side view; b) picture

The novel clamping unit consists of tempered steel grips with a corrugated surface, which are tightened with high strength bolts in a steel frame. Usually, the result of using corrugated surface steel grips would be a fractured laminate, therefore plane surfaces were used in the prestressing systems reviewed in Section 1.1. The catch of the current invention is to add epoxy adhesive to both upper and lower contact surfaces of the grips and the laminate, and tighten with high strength bolts in the steel frame (see Fig. 3.10). The result after one day of hardening is a more effective clamping unit with very low slip potential.

Fig. 3.10. Clamping of the laminate: a) bonding the laminate; b) tightening the bolts

Such clamping units can be attached on both sides of the laminate one day before strengthening outside the construction site, this way the strengthening time is not higher than using any other existing method. If one day does not make any difference, the clamping unit can be attached to one side of the laminate and the

other end can be anchored to the concrete with riffled surface steel plates (see Fig. 3.11), epoxy adhesives must be added onto concrete-laminate, laminate-steel plate contact surfaces, and the plates must be tightened to concrete with anchor bolts at once. Anchoring with plane surface plates as proposed in the prior art would not withstand similar loads, even bonded with epoxy adhesive one day before the tensioning.

Fig. 3.11. Anchor plates

The specimen preparation process is the same as with other this type techniques described in Section 1.1, which do not make additional damage to the concrete. This process includes grinding and cleaning of the concrete surface, and cleaning of the laminate (Fig. 3.12).

Fig. 3.12. Specimen preparation: a) concrete surface grinding; b) concrete surface cleaning; c) cleaning of the laminate

Considering the laminate is composed of fibres and a polymeric matrix, and its surface slip properties will be similar to those of other fibres in the polymeric matrix, it becomes obvious that the present invention can be used with other FRP laminates. It should be noted, during the research program, different angles of bending CFRP laminate in a clamping unit were tested and 7° was found as an optimal safe angle of bending laminate. There is a high risk of breaking the laminate with bending angles higher than 10° .

3.1.3. Experimental Results

As it was expected for the RC members with a balanced reinforcement ratio, the failure mode of the control beams was steel yielding followed by concrete crushing (Fig. 3.13).

Fig. 3.13. Failure modes of the control beams

All BS series beams, strengthened before external load action, ended up with steel yielding followed by interfacial FRP debonding (Fig. 3.14).

There was an initial stress-strain state and initial deflection in BSM series beams, therefore, the laminate pressed better to the beam during prestressing. However, the constant action of the external load may adversely affect the composite action of the joint between the concrete and the FRP.

Failure modes of BSM series beams were similar to BS series, although the FRP reinforcement of the non-prestressed beams performed better and higher load-carrying capacity was achieved (Fig. 3.15).

A slip occurred at the anchorage zone of the beam BSM-0-1 ($M = 59.64$ kNm), at a similar load as the failure load of beam BS-0 ($M = 57.63$ kNm). Although the FRP reinforcement was later on hooked in the anchorage zone, the concrete-FRP connection was already damaged and the failure of the beam was similar to that of the control beams, therefore the moment of slip will be considered as a failure load. Similar failure character was observed in beams BSM-17, BSM-18 and BSM-35, only the laminate was hooked at loads closer to load-carrying capacity (BSM-17: $M_{slip} = 77.40$ kNm, $M_u = 81.08$ kNm; BSM-18: $M_{slip} = 72.53$ kNm, $M_u = 78.31$ kNm; BSM-35: $M_{slip} = 82.99$ kNm, $M_u = 87.33$ kNm). Apparently, the beam BSM-0-2 had better bond characteristics as FRP interfacial debonding occurred in parallel with concrete crushing. The failure mode of the remaining beams was steel yielding followed by FRP interfacial debonding.

Fig. 3.14. Failure modes of the BS series beams

Failure modes of the strengthened beams show that the full shear connection of concrete and FRP was not ensured. The overall efficiency of the strengthening can be demonstrated by the increase in the bending moment corresponding to the yield stress of the tensile steel reinforcement (yield moment, see Fig. 3.16).

Figure 3.16 represents the difference in percent of strengthened ($M_{y,i}$) and control ($M_{y,CB}$) beam yield moments, taken from moment-deflection dependency graphs. When the beams were strengthened without prestressing, the yield moment increased by more than 20 %. After prestressing with loads close to 50 % of FRP strength, the yield moment increased by more than 100 %. The difference is a little lower for BSM series beams as the steel reinforcement had an initial stress state at the strengthening moment. This proves that prestressing is a great way to

exploit the potential of this high strength material and significantly improve the performance of the structure.

Fig. 3.15. Failure modes of the BSM series beams

Fig. 3.16. Difference in percent of strengthened and control beam yield moments: a) BS series beams; b) BSM series beams

The impact of prestressing on serviceability limit states is illustrated in Figures 3.17–3.19.

Maximum crack width development in BS series beams is presented in Fig. 3.17a and in BSM series in Fig. 3.17b.

Fig. 3.17. Crack width development: a) BS series beams; b) BSM series beams

Crack width had a high variation in not strengthened RC members, and when the elements were strengthened before loading (BS series), the crack width development was stable from the start of loading. On the other hand, when the RC members were strengthened under external load action (BSM series), a common situation in practice, crack width development was unstable until strengthening, and it was stabilized and improved after prestressing of FRP reinforcement. The concrete cracks were closed to a certain level at the moment of prestressing. However, a crack width jump was visible after the external load increment.

Mid-span deflection response of BS series beams is given in Fig. 3.18 and BSM series beams in Fig. 3.19.

Fig. 3.18. Mid-span deflection of BS series beams: a) full; b) cutout

Fig. 3.19. Mid-span deflection of BSM series beams: a) full; b) cutout

Although the BS-48 and BS-49 beams had a greater negative curvature due to prestressing, the concrete cracked during prestressing and the character of moment-deflection graphs became similar to the beams with lower prestressing levels BS-20 and BS-22, which did not crack during prestressing. However, the bending moment corresponding to the steel reinforcement yield stress (denoted as yield moment) was proportionately higher for the beams with higher prestressing stress.

As in the case of crack width, a deflection jump was observed immediately after post-prestressing external load increment of BSM series beams. To clarify this phenomenon, the strains of tensile concrete were compared with the ones of FRP at the same sections. Strain increment after decompression compared for BS series beams (Fig. 3.20–3.24) and the strain increment after post-prestressing increase of external load for BSM series beams (Fig. 3.25–3.31).

Fig. 3.20. Comparison of CFRP and tensile concrete strains of the beam BS-0

The strains of BS-0 beam were measured with 50 mm base LVDT's at the 4 largest cracks. The effect of concrete-FRP partial shear connection revealed at all four cracks. The dotted lines in the figures connect the start point of a comparison to the point of the bending moment corresponding to the yield stress of the steel reinforcement. These lines represent the linear descending decay of the bond.

Fig. 3.21. Comparison of CFRP and tensile concrete strains of the beam BS-20

The same linear descending decay of the concrete-FRP bond is visible in Fig. 3.21. Only in this case the redistribution of strain between the different cracks is observed after the steel yield point.

Fig. 3.22. Comparison of CFRP and tensile concrete strains of the beam BS-22

Fig. 3.22 clearly illustrates the linear descending decay of concrete-FRP bond.

Fig. 3.23. Comparison of CFRP and tensile concrete strains of the beam BS-48

The FRP reinforcement slip at the crack occurred at $0.43 M_u$ in the beam BS-48, and the difference between concrete and FRP strains was steadily increasing until stabilization at $0.69 M_u$ (see Fig. 3.23).

The effect of partial shear connection was not revealed at the points of the beam BS-49 where the strains were measured (see Fig. 3.24). BS-49 was the only beam where the slip in the contact surface of concrete and FRP was not recorded.

The vertical dotted line drawn in Fig. 3.25 represents the average of the ratio of the FRP and concrete strains, which approximately corresponds to the values obtained at the yield moment of the above cases.

Fig. 3.24. Comparison of CFRP and tensile concrete strains of the beam BS-49

Fig. 3.25. Comparison of CFRP and tensile concrete strains of the beam BSM-0-1

Fig. 3.26. Comparison of CFRP and tensile concrete strains of the beam BSM-0-2

The difference in concrete and FRP strain of the BSM-0-2 beam at the measured crack occurred at $0.46 M_u$ and was stable up to the yield moment at which

further linear decomposition of the bond was observed up to the failure load. Vertical mean line corresponds to the same point as in BSM-0-1 case.

Fig. 3.27. Comparison of CFRP and tensile concrete strains of the beam BSM-17

Fig. 3.28. Comparison of CFRP and tensile concrete strains of the beam BSM-18

The difference in strains immediately appeared at the measured crack of the beams BSM-17 and BSM-18 (see Fig. 3.28).

A clear descending line is recorded in the beam BSM-35 at the 50 mm and 200 mm base measuring points up to the steel yield moment, afterwards aligned with the vertical (see Fig. 3.29). Descending lines recorded in BS series and BSM-0-1, BSM-0-2, and BSM-35 beams. However, a clear vertical mean line can be drawn in all BSM series beams.

Fig. 3.29. Comparison of CFRP and tensile concrete strains of the beam BSM-35

Fig. 3.30. Comparison of CFRP and tensile concrete strains of the beam BSM-46

The difference of strains immediately appeared at the 50 mm and 200 mm base measuring points in beams BSM-46 and BSM-56, as clear vertical lines are visible in figures 3.30 and 3.31.

Fig. 3.31. Comparison of CFRP and tensile concrete strains of the beam BSM-56

To summarize, the gradual decay of the adhesive joint and the subsequent activation of the anchors are observed in most BS series beams. However, an immediate activation of the anchors is observed in most cases of the BSM series beams. This phenomenon causes a strain difference of tensile concrete and FRP reinforcement, which can be evaluated with the bond stiffness reduction factor described in Chapter 2.1.

In view of the conclusions above, an additional coefficient is introduced to the bond stiffness reduction factor:

$$\psi_f = k_{pl} \psi_{f.B} \quad (3.3)$$

Coefficient $k_{pl} = 1.00$ for the beams strengthened under external load action (BSM) and k_{pl} can be calculated assuming a linear descending decay of the adhesive joint for the BS series beams:

$$k_{pl} = 2 - \frac{E_{s1} \varepsilon_{s1}}{f_{s1,y}} \geq 1. \quad (3.4)$$

The above factors allow to evaluate concrete-FRP partial shear connection effect on the strengthened RC member's deformations.

3.2. Validation of Theoretical Models

3.2.1. Properties of the Additional Beams Used in the Analysis

The results of additional 73 RC beams strengthened with externally bonded and near surface mounted CFRP and GFRP sheets, laminates, strips, and rods were taken from various scientific publications for the load-carrying capacity and deflection analysis. Geometric and material properties of these beams are presented in Table 3.5. Attention should be paid to the percentages of the internal and external reinforcement of the specimens in Table 3.5, also, the different prestressing levels, and the different reinforcement systems used.

Additional beams for crack width analysis: two RC beams strengthened with externally bonded CFRP sheets (B3, B4) tested earlier at Vilnius Gediminas Technical University laboratory (Slaitas *et al.* 2018, Daugevičius *et al.* 2012, Daugevičius and Valivonis, 2013), presented in Fig. 3.32a and Table 3.6. Also, three additional RC beams strengthened with NSM GFRP rods (GCB series) tested by Xing *et al.* (2018). The beams presented in Fig. 3.32b and Table 3.6 were strengthened with NSM GFRP rods with different reinforcement ratios of internal and external reinforcements, also, Xing *et al.* managed to record the crack width of these beams up to element failure.

Table 3.5. Main parameters of additional beams

Reference	A_{sl}/b d , %	f_y , MPa	A_f $/bd_f$, %	f_f , MPa	E_f , GPa	σ_p , MPa	EBR/ NSM
Slaitas <i>et al.</i> 2018	0.76	569	0.37	2334	213	0–120	EBR
Peng <i>et al.</i> 2014	0.85	400	0.11	3100	165	1000	EBR
	0.85	400	0.13– 0.14	2068	131	0–1000	NSM
Jung <i>et al.</i> 2005, 2017	0.40	426	0.04– 0.12	2453– 3479	165– 230	0	EBR
	0.40	426	0.04– 0.11	1878– 2453	121– 165	0	NSM
	0.45	436	0.04– 0.22	1500– 2483	100– 167	0	NSM
Woo <i>et al.</i> 2008	0.29– 1.19	466– 501	0.08	2850	165	0–1323	EBR
Fu <i>et al.</i> 2017	0.50– 0.75	525– 531	0.11	3263	251	0	EBR
Sharaky <i>et al.</i> 2014, 2015	0.58	545	0.12– 0.26	1350– 2350	64–170	0	NSM
	0.58	540	0.13– 0.26	1350– 2500	64–170	0	NSM
El-Hacha and Gaafar 2011	0.77	475	0.08	2167	130	0–1241	NSM
Barros <i>et al.</i> 2005	0.54– 0.94	730	0.16– 0.24	2740	159	0	NSM
Rezazadeh <i>et al.</i> 2014	0.39	585	0.06	1922	164	0–823	NSM

Table 3.6. Additional beams for crack width analysis

Beam	f_{cm} , MPa	$f_{y.sl}$, MPa	E_{sl} , GPa	$f_{y.s2}$, MPa	E_{s2} , GPa	f_{fu} , MPa	E_f , GPa	σ_p , MPa
B3	28.70	318	205	420	204	4800	230	0
B4								
GCB-1	37.44*	498	203	531	204	966	40	0
GCB-2	37.84*	498	203	531	204	966	40	0
GCB-3	39.36*	505	205	531	204	966	40	0

*– derived from compressive strength of concrete cube, which was equal to 46.8, 47.3, 49.2 MPa respectively.

Fig. 3.32. Geometric characteristics: a) EBR CFRP sheets (Slaitas *et al.* 2018, Daugevičius *et al.* 2012, Daugevičius and Valivonis, 2013); b) NSM GFRP rods (Xing *et al.* 2018)

3.2.2. Comparison of the Experimental and Numerical Results

Figure 3.33 and Table 3.7 represent the comparison between the ultimate bending moments obtained experimentally and numerically.

Fig. 3.33. Experimental-calculated load carrying capacity of the strengthened beams

Table 3.7. Experimental-calculated load carrying capacity of the strengthened beams

Statistical parameters	$M_{u,exp} / M_{u,calc},$ $\psi_f = \psi_{f,B}$	$M_{u,exp} / M_{u,calc},$ $\psi_f = \psi_{f,T}$	$M_{u,exp} / M_{u,calc},$ $\psi_f = 1$
Mean	1.12	1.06	0.89
Standard deviation	0.15	0.14	0.14
Coefficient of variation, %	13.83	13.66	15.82
Coefficient of correlation, R	0.97	0.97	0.96

Ultimate bending moment with low systematic error (1.06–1.12), low random error (0.14–0.15), low coefficient of variation (13.66–13.83 %) and high coefficient of correlation (0.97) was predicted sufficiently precisely. Although the statistical difference between bending moments with the reduced FRP-concrete contact stiffness and without it was not significant, the points from the failure side moved to the safety side. Moreover, some beams achieved only a slightly higher load than 50 % of a predicted load bearing capacity without contact stiffness reduction ($\psi_f = 1$), what is critically unsafe. Therefore, the proposed calculation technique should be considered for use in practice.

The bond stiffness reduction factor $\psi_{f,B}$ in Fig. 3.33 is calculated by Eq. 2.14 accepting design bond length equal to crack spacing $L_b = L_{cr}$. The bond stiffness reduction factor $\psi_{f,T}$ is calculated by Eq. 2.19 accepting design bond length equal to the distance from the FRP reinforcement end to the mid-span of the beam $L_b = L_{bond} / 2$. The values of the ultimate bending moment obtained considering these failure modes are similar, therefore both approaches can be used for a prediction of the load carrying capacity of flexural RC members strengthened with FRP composites.

Bending moment – deflection dependency graphs of the beam series BS and BSM presented in figures 3.34 and 3.35. Deflections denoted as *Gross* and *Cracked* were adopted in the following way.

$$\delta_{Gross} = \frac{\xi M_a L^2}{E_{cm} I_g} - \frac{PL^2 e_p}{8E_{cm} I_g}; \quad \delta_{Cracked} = \frac{\xi M_a L^2}{E_{cm} I_{cr}} - \frac{PL^2 e_p}{8E_{cm} I_{cr}}. \quad (3.5)$$

Comparison of experimental and numerical graphs confirms the following assumptions: (1) if the element is already cracked before loading, the cracked and not cracked zones of the beam distributes approximately equally along the element, i.e. $(M_{cr} / M_a)^m \leq 0.5$. (2) When RC members were strengthened from the start of loading without prestressing (see Fig. 3.36), the effect of FRP reinforcement should be not taken into account for the deflection response up to the bending moment corresponding to the steel yield stress. (3) The tension stiffening effect evaluation power $m \approx 1.00$ while BS series beams had a negative curvature due to prestressing, afterwards it is in between $1.00 \leq m \leq 2.00$. However, power

$m \approx 1.00$ for BSM series beams and BS series beams, which did not crack during prestressing. (5) Correspondence between experimental and numerical graphs proves that the slip between concrete and FRP exists and its effect can be evaluated sufficiently precisely by the method proposed in this thesis.

Fig. 3.34. Bending moment-deflection dependency graphs: a) BS-0; b) BS-20; c) BS-22; d) BS-48; e) BS-49

Moment-deflection graphs of the beams BSM-17 and BSM-46 do not represent the accuracy of the proposed method for strengthened members, therefore these beams were additionally evaluated from the strengthening point (see Fig. 3.37).

Fig. 3.35. Bending moment-deflection dependency graphs: a) BSM-0-1; b) BSM-0-2; c) BSM-17; d) BSM-18; e) BSM-35; f) BSM-46; g) BSM-56

Fig. 3.36. Bending moment-deflection dependency graph of BS-0 beam

Fig. 3.37. Deflection from the strengthening point: a) BSM-17; b) BSM-46

Since the break point in the moment-deflection graph is the bending moment corresponding to the steel yield stress, an accurate prediction of it is important. Comparison of numerical (taking into account the assumptions above) and experimental mid-span deflection at this point with additional beams taken from various scientific publications (Table 3.4) presented in Fig. 3.38.

Fig. 3.38. Deflection corresponding to yield moment: a) dependence of numerical and experimental results; b) error graph

Although in some cases the numerical results were more accurate while assuming power $m = 1.00$, the overall statistics show that a more accurate result is obtained (or with reserve) with $m = 2.00$. However, a novel experimental research

has shown the opposite, that the power $m \approx 1.00$ for the beams strengthened under external load action (BSM series) and when prestressing did not cause flexural cracks for BS series beams.

It should also be noted that the highest errors were obtained in beams without prestressing, which only confirmed the theory that the deflection of such beams should be calculated without evaluation of the FRP work.

The flexural stiffness of the element decreases and the deflection law changes after the steel reinforcement reaches the yield stress. Finding a further deflection increment is important for strengthened structures as the deflection does not yield. Therefore, the comparison of the numerical and experimental values of mid-span deflection corresponding to load-carrying capacity of the element is presented in Fig. 3.39.

Fig. 3.39. Deflection corresponding to load-carrying capacity: a) dependence of numerical and experimental results; b) error graph

Mid-span deflection with a low systematic error (1.11 for δ_y and 1.04 for δ_u), low random error (0.19 for δ_y and 0.24 for δ_u), low coefficient of variation (17.00 % for δ_y and 23.42 % for δ_u) was predicted sufficiently precisely. It was observed that even small levels of prestressing (especially when prestressing do not cause flexural cracks) engage the external reinforcement and greatly improve the deflection response.

Bending moment-crack width dependency graphs of the beam series BS and BSM are presented in Figures 3.40 and 3.41.

Predicted bending moment-crack width dependency graphs sufficiently precisely replicate the experimental points (coefficient of correlation 0.95÷1.00). Crack width of the beams BSM-46 and BSM-56 was predicted with higher reserve – the effect of prestressing was higher for the beam BSM-56 and the concrete-FRP partial shear connection effect was lower for both beams.

Fig. 3.40. Bending moment-crack width dependency graphs a) BS-22; b) BS-20; c) BS-49; d) BS-48; e) BS-0; f) error graph

Fig. 3.41. Bending moment-crack width dependency graphs a) BSM-0-1; b) BSM-0-2; c) BSM-17; d) BSM-18; e) BSM-35; f) BSM-46; g) BSM-56; h) error graph

Bending moment-crack width dependency graphs of additional beams (Table 3.6) presented in Fig. 3.42.

Fig. 3.42. Bending moment-crack width dependency graphs: a) B3 and B4 (Slaitas *et al.* 2018, Daugevičius *et al.* 2012, Daugevičius and Valivonis, 2013); b) GCB-1; c) GCB-2; d) GCB-3 (Xing *et al.* 2018)

Compliance of the calculated and numerical results of additional beams in Fig. 3.42 confirm the theory that the crack width can be predicted sufficiently precisely, even after the steel yield point.

3.3. Conclusions of Chapter 3

Within this Chapter, the following conclusions can be drawn:

1. Different angles of bending CFRP laminate in a clamping unit were tested and 7° was found as an optimal safe angle. There is a high risk of breaking the laminate with bending angles higher than 10° .
2. The tempered steel plates profiled with peaks are found as the best solution for preventing FRP-clamping unit interfacial slip. Larger area anchor plates can be profiled with linear grooves. Additional layer of epoxy resin between the surfaces prevents the disintegration of the laminate polymer matrix.
3. When the RC elements were strengthened before loading (BS series), the crack width development was stable from the concrete cracking point. On

the other hand, when the RC members were strengthened under external load action (BSM series), a common situation in practice, the crack width development was unstable until strengthening, and it was stabilized and improved after prestressing of FRP reinforcement. The concrete cracks were closed to a certain level during prestressing. However, a crack width jump was recorded after the external load increment.

4. When the beams were strengthened without prestressing, the yield moment has increased by more than 20% from the control beams. After prestressing with loads close to 50% of FRP strength, the yield moment has increased by more than 100% from the control beams. Accordingly, the deflection and crack widths were also reduced. The difference is a little lower for BSM series beams as the steel reinforcement was subjected to an initial stress state at the strengthening moment. This proves that prestressing is a great way to exploit the potential of this high strength material and significantly improve the performance of the structure.
5. The gradual decay of the adhesive joint and the subsequent activation of the anchors are observed in most BS series beams. However, an immediate activation of the anchors is observed in most cases of the BSM series beams. This phenomenon causes a strain difference of tensile concrete and FRP reinforcement, which can be evaluated by the proposed bond stiffness reduction factor.
6. Ultimate bending moment with low systematic error (1.06–1.12), low random error (0.14–0.15), low coefficient of variation (13.66–13.83 %) and high coefficient of correlation (0.97) was predicted sufficiently precisely. Although the statistical difference between bending moments with the reduced FRP-concrete contact stiffness and without it was not significant, the points from the failure side moved to the safety side. Moreover, some beams achieved only a slightly higher load than 50 % of a predicted load bearing capacity without contact stiffness reduction ($\psi_f = 1$), what is critically unsafe. Both the proposed numerical techniques considering the intermediate crack-induced debonding failure mode and the simplified bond strength solution are acceptable for use.
7. When RC members are strengthened before external load action without prestressing, the effect of FRP reinforcement should be not taken into account for the deflection response up to the bending moment corresponding to the steel yield stress.
8. Correspondence between numerical and experimental moment-deflection graphs ($R = 0.98$ –1.00) proves that the slip between concrete and FRP exists and its effect can be evaluated sufficiently precisely by the method proposed in this thesis.

9. It was observed that even small levels of prestressing (especially when prestressing do not cause flexural cracks) engage the external reinforcement and greatly improve the deflection response.
10. The comparison of numerical and experimental results of the strengthened beams tested by other authors revealed that the numerical results were more accurate while assuming the tension stiffening effect evaluation power $m = 2.00$. However, a novel experimental research has shown the opposite, that the power $m \approx 1.00$ for the beams strengthened under external load action (BSM series) and when prestressing did not cause flexural cracks for BS series beams.
11. Predicted bending moment-crack width dependency graphs sufficiently precisely replicate the experimental points (coefficient of correlation $0.95 \div 1.00$). Moreover, the results of additional beams tested by other authors confirmed the theory that the crack width can be predicted sufficiently precisely, even after the steel yield point.

General Conclusions

Summarizing the findings of the dissertation, the following conclusions can be drawn:

1. Existing FRP prestressing systems use plain surfaces for clamping units, leading to a risk of slip in a contact surface, or additional damage to the concrete is required, which can be impossible due to the risk of damaging internal reinforcements. The significance of the proposed system is to use tempered steel plates profiled with peaks for preventing FRP-clamping unit interfacial slip. Larger area anchor plates can be profiled with linear grooves. Additional layer of epoxy resin between the surfaces prevents the disintegration of the laminate polymer matrix.
2. Intermediate crack induced (IC) debonding is distinguished as the most common failure mode of FRP strengthened RC members. Therefore, methods assessing this type of failure are also applied in design codes. However, techniques found in the literature use the fracture energy/failure criterion or limit stresses derived from narrow parametric scope pull-off/out shear tests and usually work only for EBR or only for NSM strengthening techniques. Some research has shown that even models distinguished elsewhere as having the highest level of accuracy can have high errors when the test parameters are changed.

3. Partial shear connection of concrete and FRP reinforcement can be taken into account by reducing the axial stiffness of FRP reinforcement with the reduction factor based on built-up bars theory. However, the application of this theory is greatly influenced by the finding of the effective shear modulus of the joint. Therefore, the analytical solution is proposed comprising an intermediate crack-induced debonding failure mode and a simplified bond strength solution.
4. The gradual decay of the adhesive joint and the subsequent activation of the anchors are observed in most BS series beams. However, an immediate activation of the anchors is observed in most cases of the BSM series beams. This phenomenon causes a strain difference of tensile concrete and FRP reinforcement, which can be evaluated by the proposed bond stiffness reduction factor.
5. Load bearing capacity of the RC beams strengthened with externally bonded and near surface mounted CFRP and GFRP laminates, sheets, strips, and rods with low systematic error (1.06–1.12), low random error (0.14–0.15), low coefficient of variation (13.66–13.83 %) and high coefficient of correlation (0.97) was predicted sufficiently precisely. If the load carrying capacity was calculated assuming concrete-FRP full shear connection ($\psi_f = 1$), some beams would fail at a slightly higher load than 50 % of a predicted load carrying capacity. Therefore, both the proposed partial shear connection evaluation techniques considering the intermediate crack-induced debonding failure mode and the simplified bond strength solution are acceptable for use.
6. The direct relation of concrete crack parameters and strains is proposed based on the modified fracture mechanics crack model. The mathematically and physically correct application of the proposed relation is presented in two possible ways: prediction of the maximum crack width development and stress-strain state evaluation – reverse task. Predicted bending moment-crack width dependency graphs sufficiently precisely replicate the experimental points (coefficient of correlation 0.95÷1.00). Moreover, the results of additional beams tested by other authors confirmed the theory that the crack width can be predicted sufficiently precisely, even after the steel yield point.
7. A deflection calculation technique based on the modelling of cracked and uncracked cross-sections with springs in series and incorporating the effects of prestressing and concrete-FRP partial shear connection is proposed. Correspondence between experimental and numerical graphs ($R = 0.98 \div 1.00$) proves that the slip between concrete and FRP exists and its effect can be evaluated sufficiently precisely by the method proposed in this paper.

8. It was observed that even small levels of prestressing (especially when prestressing do not cause flexural cracks) engage the external reinforcement and greatly improve the deflection response. When RC members are strengthened before external load action without prestressing, the effect of FRP reinforcement should be not taken into account for the deflection response up to the bending moment corresponding to the steel yield stress.

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List of Scientific Publications by the Author on the Topic of the Dissertation

Papers in the Reviewed Scientific Journals

Šlaitas, J.; Valivonis, J. 2021. Full moment-deflection response and bond stiffness reduction of RC elements strengthened with prestressed FRP materials, *Composite Structures* 260:1–13. Oxford: Elsevier. DOI: 10.1016/j.compstruct.2020.113265. [Clarivate Analytics Web of Science, IF: 5.138].

Šlaitas, J.; Valivonis, J. 2021. Concrete cracking and deflection analysis of RC beams strengthened with prestressed FRP reinforcements under external load action, *Composite Structures* 255:1–11. Oxford: Elsevier. DOI: 10.1016/j.compstruct.2020.113036. [Clarivate Analytics Web of Science, IF: 5.138].

Šlaitas, J.; Valivonis, J.; Rimkus, L. 2020. Evaluation of stress-strain state of FRP strengthened RC elements in bending. Fracture mechanics approach, *Composite Structures* 233:1–9. Oxford: Elsevier. DOI: 10.1016/j.compstruct.2019.111712. [Clarivate Analytics Web of Science, IF: 5.138].

Šlaitas, J.; Daugevičius, M.; Valivonis, J.; Grigorjeva, T. 2018. Crack width and load-carrying capacity of RC elements strengthened with FRP, *International Journal of Polymer Science* 2018:1–14. London: Hindavi. DOI: 10.1155/2018/6274287. [Clarivate Analytics Web of Science, IF: 1.892].

Šlaitas, J.; Valivonis, J.; Juknevičius, L.; Šalna, R. 2018. Load-bearing capacity of flexural reinforced concrete members strengthened with fiber-reinforced polymer in fracture stage, *Advances in Mechanical Engineering* 10(5):1–16. London: SAGE publications. DOI: 10.1177/16878140187729. [Clarivate Analytics Web of Science, IF: 1.024].

Šlaitas, J.; Hlavac, Z.; Šneideris, A. 2017. Flexural reinforced concrete elements normal section bearing capacity evaluation in fracture stage, *Engineering Structures and Technologies*. 2017, Vol. 9, no. 2, p. 70–78. Vilnius; London: Technika; Taylor & Francis. DOI: 10.3846/2029882X.2017.1322919.

Šlaitas, J. 2017. Lenkiamųjų gelžbetoninių elementų, sustiprintų pluoštu armuotu kompozitu, laikomosios galios skaičiavimas pagal kritinį plyšio aukštį = Flexural reinforced concrete elements, strengthened with fibre reinforced polymer, bearing capacity evaluation according to limit crack depth, *Mokslas – Lietuvos ateitis = Science – Future of Lithuania: Statyba, transportas, aviacinės technologijos = Civil and Transport Engineering, Aviation Technologies* 9(5):507–519. Vilnius: Technika. DOI: 10.3846/mla.2017.1079.

Papers in Other Editions

Šlaitas, J.; Valivonis, J. 2019. Crack parameters in normal section of FRP strengthened RC elements, *Proceedings of the 13th International Conference Modern Building Materials, Structures and Techniques* 2019:475–482. Vilnius: Technika. DOI: 10.3846/mbmst.2019.028.

Šlaitas, J.; Valivonis, J. 2018. Fracture mechanics based calculation model of flexed RC elements strengthened with FRP, *Proceedings of the 9th international conference on Fibre-Reinforced Polymer (FRP) composites in Civil Engineering (CICE 2018)*. 17–19 July 2018, Paris, France. Part 2, 548–556.

Patent applications

Šlaitas, J.; Valivonis, J. 2021. Device and method for reinforcing structures with prestressed fibre-reinforced-polymer laminates, *European patent application* No. EP21167213.4.

Summary in Lithuanian

Įvadas **Problemos formulavimas**

Pastaraisiais dešimtmečiais pluoštu armuotų kompozitinių (FRP) medžiagų panaudojimas gelžbetoninių konstrukcijų stiprinimui nuolat auga. Konstrukcijų stiprinimo priežastimi gali būti projektavimo ir statybos klaidos, prasta konstrukcijų priežiūra bei medžiagose vykstantys korozijos ir senėjimo procesai. Daugeliu atvejų FRP medžiagų panaudojimas yra racionalus stiprinimo sprendimas. Lyginant su tradiciniais stiprinimo metodais šios medžiagos yra lengviau tvirtinamos, tai leidžia sumažinti darbo sąnaudas, jos turi geresnę estetinę vaizdą bei yra ilgaamžiškesnės. Be to, šis kompozitas gali būti efektyviai panaudotas jį iš anksto įtempiant. Tokiu būdu ne tik padidinama stiprinamo elemento laikomoji galia, bet ir sumažinami įlinkiai ir padidinamas atsparumas pleišėjimui. Tačiau dažniausiai pasitaikantis sustiprintų konstrukcijų irimo atvejis yra priešlaikinis trapus betono ir kompozito jungties irimas. Kol gelžbetoninė konstrukcija nesupleišėjusi betono ir kompozito jungtyje šlyties deformacijos nepasireiškia ir galima teigti, kad jungtis yra standi. Betone atsivėrus plyšiams, plyšio kraštuose susidaro įtempių koncentracija, tai sumažina betono ir kompozito jungties standumą, ir sukelia praslydimą, kuris iššaukia betono ir kompozito jungties trapų irimą. Tokie irimo atvejai dažnai pasitaiko kompozitu sustiprintuose lenkiamuose elementuose. Kitų autorių pasiūlytuose betono ir kompozito jungties stiprumo vertinimo modeliuose naudojamos irimo kriterijų ar irimo energijos reikšmės išvestos iš mažos parametrinės sklaidos kompozito atplėšimo bandymų, nors jungties elgsena ir irimo kriterijai gali būti visai kitokie realaus dydžio lenkiamuose elementuose

su gerokai didesniais kompozito tvirtinimo ilgiais. Dažnai skaičiuojant kompozitu sustiprintų gelžbetoninių elementų deformacijas ir pleišėjimą tarp betono ir kompozito vertinama standi jungtis.

Darbo aktualumas

Atsižvelgiant į esamą situaciją, atsiranda analitinių laikomosios galios, įlinkio, plyšio parametrų skaičiavimo metodų, vertinančių betono ir kompozito dalinai standžios jungties ir kompozito išankstinio įtempimo efektus, poreikis. Tokiu būdu būtų ne tik efektyviai įvertinamas galimas priešlaikinis elemento irimas, bet ir aprašoma pilna dalinai standžios jungties tarp betono ir kompozito elgsena ir jos įtaka elemento deformacijoms ir pleišėjimui.

Nors stiprinimas dažniausiai reikalingas esamoms eksploatuojamoms konstrukcijoms, tačiau betono ir kompozito jungties darbas, kai sijos stiprinamos veikiant išorinei apkrovai, iki šiol pilnai nebuvo ištirtas. Todėl nauji betono ir kompozito dalinai standžios jungties elgsenos ir sustiprinto gelžbetoninio elemento deformacijų duomenys turėtų padaryti įtaką tolimesniems tokių konstrukcijų tyrimams ir praktiniam kompozitinių medžiagų panaudojimui.

Kadangi kompozitinių medžiagų išankstinis įtempimas stiprinant eksploatuojamas konstrukcijas kelia nemažai iššūkių statybos inžinieriams, atsirado esamo technikos lygio analizės ir naujų kompozitinės medžiagos inkaravimo sprendinių poreikis.

Tyrimo objektas

Tyrimų objektas yra trumpalaikė apkrova veikiamų iš anksto įtemptu anglies pluošto kompozitu sustiprintų, lenkiamųjų gelžbetoninių elementų betono ir kompozito dalinai standžios jungties įtakos betono pleišėjimui, įlinkiui ir laikomajai galiai vertinimas.

Darbo tikslas

Disertacijos tikslas yra sukurti iš anksto įtemptu anglies pluošto kompozitu sustiprintų lenkiamųjų gelžbetoninių elementų išankstinio įtempimo jėgos, betono ir anglies pluošto kompozito dalinio jungties standumo įtakos betono pleišėjimui, įlinkiui ir laikomajai galiai vertinimo modelį.

Darbo uždaviniai

Darbo tikslui pasiekti sprendžiami šie uždaviniai:

1. Sukurti sijų laikomosios galios, įlinkių ir betono plyšio parametrų vertinimo modelius, įvertinant anglies pluošto kompozito išankstinio įtempimo bei betono ir kompozito dalinai standžios jungties efektus.
2. Sukurti anglies pluošto kompozito išankstinio įtempimo įrangą ir pasiūlyti ją palydinčią stiprinimo technologiją.
3. Atlikti gelžbetoninių sijų, sustiprintų iš anksto įtemptu anglies pluošto kompozitu, eksperimentinius tyrimus, vertinant betono plyšio parametrų ir sijos įlinkio vystymąsi.

4. Palyginti eksperimentinius ir teorinius rezultatus. Remiantis gautais rezultatais patikslinti skaičiavimo metodus.

Tyrimų metodika

Nagrinėjant darbo objektą taikyti šie metodai:

- teoriniai (analizės bei sintezės) metodai: sukurtas iš anksto įtemptu CFRP kompozitu sustiprintų gelžbetoninių elementų laikomosios galios, įlinkio ir plyšio pločio vertinimo teorinis modelis;
- eksperimentiniai metodai: atliktas gelžbetoninių sijų, sustiprintų iš anksto įtemptomis CFRP juostomis, eksperimentinis tyrimas;
- statistikos metodai: daromos išvados renkant, analizuojant ir paaiškinant statistinius duomenis.

Darbo mokslinis naujumas

1. Pasiūlytas naujas analitinis betono ir CFRP kompozito dalinai standžios jungties vertinimo modelis.
2. Pasiūlyti nauji alternatyvūs laikomosios galios, įlinkio ir pleišėjimo skaičiavimo modeliai, įvertinant kompozito išankstinio įtempimo bei betono ir CFRP kompozito dalinai standžios jungties efektus.
3. Pasiūlyta nauja FRP juostų išankstinio įtempimo įranga su efektyviais rifliuotais inkaravimo įrangos paviršiais.
4. Gauti nauji betono ir CFRP kompozito dalinai standžios jungties elgsenos eksperimentiniai duomenys.
5. Gauti nauji trumpalaikė apkrova veikiamų gelžbetoninių sijų, sustiprintų iš anksto įtemptu anglies pluošto kompozitu, eksperimentinių tyrimų rezultatai.

Darbo rezultatų praktinė reikšmė

Šiame darbe siūloma analitinis betono ir kompozito dalinai standžios jungties vertinimo modelis, įvertinant pilną jungties irimą nuo betono pleišėjimo iki pilno elemento suirimo, taip pat gelžbetoninių elementų, sustiprintų iš anksto įtemptomis FRP juostomis, laikomosios galios, įlinkio ir plyšio pločio skaičiavimo modeliai. Siūloma unikali išoriškai klijuojamų FRP juostų išankstinio įtempimo įranga su efektyvia inkaravimo sistema. Atlikti eksperimentiniai tyrimai leido išanalizuoti betono ir kompozito dalinai standžios jungties bei kompozito išankstinio įtempimo įtaką sustiprinto elemento elgsenai. Buvo išspręsti teoriniai ir praktiniai tinkamo sustiprintų konstrukcijų darbo vertinimo ir išankstinio įtempimo technologijos klausimai.

Ginamieji teiginiai

1. Darbe pasiūlytas analitinis betono ir anglies pluošto kompozito dalinai standžios jungties skaičiavimo modelis leidžia įvertinti betono ir kompozito deformacijų skirtumą bei jungties standumo sumažėjimą.

2. Darbe pasiūlyti laikomosios galios, įlinkio ir plyšio pločio skaičiavimo modeliai, įvertinantys betono ir anglies pluošto kompozito dalinai standžios jungties bei kompozito išankstinio įtempimo efektus, leidžia pakankamai tiksliai įvertinti iš anksto įtemptu anglies pluošto kompozitu sustiprintų lenkiamųjų gelžbetoninių konstrukcijų tinkamumo ir saugos ribinius būvius.
3. Lenkiamųjų anglies pluošto kompozitu sustiprintų gelžbetoninių elementų plyšio plotis gali būti tiesiogiai susietas su elemento deformacijomis panaudojant irimo mechanika paremtą plyšio modelį.
4. Norint gauti tikslesnius įlinkio ir plyšio pločio skaičiavimo rezultatus, turi būti įvertinta betono ir kompozito dalinai standi jungtis. Jungties elgsena ir elemento tempiamasis sustandėjimas skiriasi, kai elementas stiprinamas veikiant išorinei apkrovai.

Darbo rezultatų aprobavimas

Disertacijos tema paskelbti 9 moksliniai straipsniai, iš kurių 5 – žurnaluose turinčiuose cituojamumo rodiklį. Disertacijoje atliktų tyrimų rezultatai buvo paskelbti 4-iose tarptautinėse ir 1-oje nacionalinėje konferencijose:

- 23-oji tarptautinė konferencija *23rd International Conference on Composite Structures (ICCS23) & 6th International Conference on Mechanics of Composites (MECHCOMP6)*, Porto, Portugalija, 2020;
- 5-oji tarptautinė konferencija *5th International Conference on Mechanics of Composites (Mechcomp 5)*, Lisabona, Portugalija, 2019;
- 13-oji tarptautinė konferencija *13th International Conference „Modern Building Materials, Structures and Techniques“ (MBMST 2019)*, Vilnius, Lietuva, 2019;
- 9-oji tarptautinė konferencija *9th International Conference on Fibre-Reinforced Polymer Composites in Civil Engineering (CICE 2018)*, Paryžius, Prancūzija, 2018;
- 20-oji Jaunųjų mokslininkų konferencija *Mokslas – Lietuvos ateitis*, Statybos inžinerija, Vilnius, Lietuva, 2017.

Disertacijos struktūra

Darbą sudaro įvadas, trys pagrindiniai skyriai, išvados, literatūros sąrašas ir autoriaus publikacijų disertacijos tema sąrašas.

Padėka

Disertacijos autorius nuoširdžiai dėkoja savo vadovui, Gelžbetoninių konstrukcijų ir geotechnikos katedros vedėjui prof. dr. Juozui Valivoniui už lyderystę, motyvaciją, neįkainojamą patirtį ir žinias.

Ypatingas dėkingumas skiriamas Arūnui Speičiui, Antanui Patapavičiui, Kęstučiui Paltanavičiui iš Konstrukcijų modelių laboratorijos už neįkainojamą pagalbą gaminant

išankstinio įtempimo įrangą. Taip pat dr. Aleksandr Sokolov, Statybinių konstrukcijų ir geotechnikos laboratorijos vedėjui Gintarui Jurkėnui, technikui Aleksandrui Zajacui, laborantams Juozui Gervei ir Grigorij Kuksov už pagalbą atliekant eksperimentinius tyrimus. Be abejo, ypatinga padėka skiriama Statybos fakulteto dekanui prof. dr. Algirdui Juozapaičiui už pasitikėjimą ir palaikymą.

Be to, autorius reiškia didžiausią padėką doc. dr. Arnoldui Šneideriui, kuris visada pasirengęs padėti, prof. dr. Dariui Bačinskui, prof. habil. dr. Gintariui Kaklauskui, prof. dr. Romualdai Kliukui ir dr. Mantui Atučiai už jų atsiliepimus ir pastabas, padėjusias patbulinti disertaciją.

Darbo autorius reiškia padėką Lietuvos mokslo tarybai ir COST asociacijai už finansinę paramą ir galimybę studijuoti užsienyje. Taip pat S&P Polska ir UAB „Delta Nova“ už CFRP juostas ir epoksidinius klijus.

Pagaliau autorius širdingai dėkoja savo šeimai, tėvams, Alvydui ir Irenai, o labiausiai mylinčiai ir palaikančiai žmonai Mirenai ir dukrai Olivijai, jūs, mergaitės, esate mano gyvenimo varomoji jėga.

1. Kompozitinėmis medžiagomis sustiprintų gelžbetoninių elementų elgsenos modelių ir irimo pobūdžio apžvalga

Pirmajame disertacijos skyriuje atlikta esamų kompozitinėmis medžiagomis sustiprintų gelžbetoninių elementų laikomosios galios, betono ir kompozito dalinai standžios jungties, įlinkių, plyšio pločio skaičiavimo modelių apžvalga. Analizė paremta esamais eksperimentiniais ir teoriniais tyrimais. Taip pat aptiriamos tipinės FRP išankstinio įtempimo sistemos, skirtos konstrukcijų stiprinimui. Skyriaus pabaigoje apibrėžtas disertacijos tikslas bei iškelti darbo uždaviniai.

Dauguma eksperimentinių tyrimų baigiasi priešlaikiniu jungties tarp betono ir kompozito irimu, todėl apžvelgiami jungties stiprumo ir standumo vertinimo metodai. Betono ir kompozito jungties irimas ties plyšiu betone išskiriamas kaip labiausiai tikėtinas tokios konstrukcijos irimo atvejis, bet literatūroje pateikiami skaičiavimo metodai naudoja irimo kriterijus/ irimo energiją ar ribinius įtempius, išvestus iš mažos parametrinės apimties kompozito atplėšimo bandymų ir dažniausiai tinka tik išoriškai tvirtinamoms FRP juostoms arba FRP armatūrai, tvirtinamai betono paviršiaus grioveluose. Taip pat apžvelgti klasikinių įlinkių ir plyšio pločio skaičiavimo metodų ypatumai.

2. Kompozitinėmis medžiagomis sustiprintų gelžbetoninių elementų saugos ir tinkamumo ribinių būvių nustatymas

Antrajame darbo skyriuje pateiktas autoriaus sukurtas modelis, leidžiantis analitiniu būdu įvertinti betono ir FRP kompozito dalinai standžią jungtį, taip pat pateikti siūlomi laikomosios galios, įlinkių ir plyšio parametrų skaičiavimo modeliai.

Grafinė betono ir FRP kompozito dalinai standžios jungties iliustracija pateikta S2.1 paveiksle.

S2.1 pav. Elemento deformacijos, įvertinant betono ir FRP kompozito dalinai standžią jungtį

S2.1 paveiksle pavaizduotą betono ir kompozito deformacijų santykį $\psi_f = \varepsilon_f / \varepsilon_{ct}$, galima pavadinti betono ir FRP kompozito jungties standumą mažinančiu koeficientu, kuris gali būti apskaičiuojamas panaudojant sudėtinių strypų teoriją. Elementui, apkrautam dviem sutelktomis jėgomis (S2.2 pav.):

$$\psi_{f.B} = 1 - \frac{\cosh(\sqrt{\alpha\beta}(0,5L - L_s))}{\sqrt{\alpha\beta}L_s \cosh(0,5\sqrt{\alpha\beta}L)} \sinh(\sqrt{\alpha\beta}L_s); \quad (S2.1)$$

$$\alpha = \frac{G_{eff} u_f}{a}; \quad \beta = \frac{1}{E_f A_f} + \frac{1}{E_c A_{c,eff}} + \frac{a^2}{EI}, \quad (S2.2)$$

čia G_{eff} – efektyvusis betono ir kompozito jungties šlyties modulis; u_f – betono ir kompozito jungties plotis; E_f , E_c – kompozito ir betono deformacijų moduliai; $A_{c,eff}$ – efektyvusis supleišėjusio skerspjūvio betono plotas (stačiakampiam skerspjūviui $A_{c,eff} = bx_c$); b – sijos plotis; x_c – betono gniuždomos zonos aukštis; EI – dviejų elementų lenkiamasis standis:

$$EI = E_c I_{c,eff} + E_f I_f \approx E_c I_{c,eff} = E_c \frac{bx_c^3}{3}. \quad (S2.3)$$

S2.2 pav. Sijų schema ir jungties modelis: a) sijos apkrovimo dviem sutelktomis jėgomis schema; b) betono ir FRP kompozito jungties modelis

Taikant sudėtinių strypų teoriją atsimušama į efektyvųjį betono ir kompozito jungties šlyties modulį (G_{eff}), kuris literatūroje dažniausiai randamas iš kompozito atplėšimo bandymų. Tačiau, kai nagrinėjama jungtis tarp dviejų plyšių, daroma prielaida, kad jungties ilgis bus lygus atstumui tarp plyšių $L_b \approx L_{cr}$, o jungties deformacija tarp dviejų plyšių bus

apytiksliai lygi plyšio pločiui $\Delta z \approx w_{cr}$. Įvertinus šias prielaidas, pasiūlytas analitinis sprendinys, kurio pagrindą sudaro medžiagų mechanika paremta efektyviojo šlyties modulio išraiška.

$$G_{eff} = \frac{\tau_{zy}}{\gamma_{zy}} = \frac{T/A_b}{\Delta z/a} = \frac{Ta}{A_b \Delta z} \approx \frac{Ta}{u_f L_{cr}^2 \varepsilon_N}. \quad (S2.4)$$

Į (S2.4) lygtį įvedus bendrosios ašinės jėgos sukeltą jungties santykinę deformaciją ir didžiausią galimą jungtimi perduodamą jėgą, efektyvusis šlyties modulis įgauna tokią išraišką:

$$G_{eff.B} \approx \frac{k_a k_L k_b A_f y_f^2 a (E_{cm} A_{ct,eff} + E_f A_f)}{u_f I_{cr} (\min(L_b, L_{b,eff}))^2}, \quad (S2.5)$$

čia k_a , k_L ir k_b – koeficientai, kuriais atitinkamai įvertinamas kompozito inkaravimas, jungties ilgis ir plotis; I_{cr} – supleišėjusio gelžbetoninio elemento inercijos momentas; y_f – atstumas tarp kompozito svorio centro ir neutraliosios ašies; $L_{b,eff}$ – efektyvusis jungties ilgis; $A_{ct,eff}$ – efektyvusis tempiamo betono, gaubiančio armatūrą plotas.

Nagrinėjant tempiamą elementą gautas eksponentinis šlyties įtempių pasiskirstymas atitinka kitų autorių eksperimentiškai nustatytas įtempių pasiskirstymo formas, o jungties standumo mažinimo koeficientas šiuo atveju:

$$\psi_{f.T} = 1 - e^{-\sqrt{\alpha\beta} L_b}. \quad (S2.6)$$

Betono ir kompozito dalinai standžios jungties vertinimo koeficientai (S2.1) ir (S2.6) lygtyse atitinkamai apibūdina jungties irimą tarp dviejų plyšių ir juostos galo atplėšimą. Abu šie irimo atvejai gali būti įvertinti klasikiniuose gelžbetonio analizės lygtyse, priėmus S2.1 paveiksle pateiktą deformacijų pasiskirstymą statmenajame pjūvyje. Tokiu atveju lenkiamojo elemento laikomoji galia:

$$M_u = A_{s1} f_{s1,y} \left(d - \frac{\lambda x_{c,u}}{2} \right) + A_{s2} \sigma_{s2} \left(\frac{\lambda x_{c,u}}{2} - d_2 \right) + A_f \psi_f \sigma_f \left(d_f - \frac{\lambda x_{c,u}}{2} \right). \quad (S2.7)$$

Bendruoju atveju sustiprinto elemento lenkimo momento-kreivio diagrama gali būti suskirstyta į 5 stadijas: 1) iki plyšio betone atsivėrimo $M_a \leq M_{cr}$; 2) nuo plyšio atsivėrimo iki elemento stiprinimo $M_{cr} < M_a \leq M_0$; 3) stiprinimo, FRP kompozito įtempimo $M_a = M_0$; 4) nuo stiprinimo iki takumo įtempių plieninėje armatūroje $M_0 < M_a \leq M_y$; 5) iki elemento suirimo $M_y < M_a \leq M_u$.

Tokiu atveju bendroji kreivio išraiška:

$$\kappa = \kappa_0 - \Delta \kappa_p + \Delta \kappa_y + \Delta \kappa_u = \frac{M_0 - M_p \psi_f}{EI_0} + \frac{M_y - M_0}{EI_y} + \frac{M_a - M_y}{EI_u}. \quad (S2.8)$$

Nustatant elemento įlinkius, didžiausią reikšmę turi tinkamas elemento lenkiamojo standžio kiekvienoje stadijoje įvertinimas. Supleišę ir nesupleišę skerspjuviai kiekvienoje stadijoje buvo modeliuojami kaip nuosekliai sujungtos spyruoklės:

$$EI_i = \frac{EI_{cr,i}}{1 - \left(\frac{M_{cr}}{M_a} \right)^m \left(1 - \frac{EI_{cr,i}}{E_{cm} I_g} \right)}. \quad (S2.9)$$

O atskiri supleišęję skerspjūviai – kaip lygiagrečiai sujungtos spyruoklės:

$$EI_{cr,y} = E_{cm}(\epsilon_c) \frac{bx_c^3}{3} + E_{s1} A_{s1} (d - x_c)^2 + \quad (S2.10)$$

$$(E_{s2} - E_{cm}(\epsilon_c)) A_{s2} (x_c - d_2)^2 + \psi_f E_f A_f (d_f - x_c)^2.$$

$$EI_{cr,u} = E_{cm}(\epsilon_c) \frac{bx_c^3}{3} + (E_{s2} - E_{cm}(\epsilon_c)) A_{s2} (x_c - d_2)^2 + \psi_f E_f A_f (d_f - x_c)^2. \quad (S2.11)$$

Betono deformacijų modulis $E_{cm}(\epsilon_c)$ įvertintas, taikant netiesinę Eurokodo 2 įtempių-deformacijų priklausomybę.

Plyšio parametrų nustatymui panaudotas kietųjų kūnų irimo mechanika paremtas plyšio modelis (Jokūbaitis *et al.* 2013, Jokūbaitis and Juknevičius, 2013). S2.3a paveiksle įvestas FRP kompozito efektas ir plyšio modelis suligintas su deformacijų profiliu (S2.3b pav.).

S2.3 pav. Plyšio parametrų ir deformacijų ryšys: a) plyšio modelis; b) deformacijų profilis

Iš trikampių 4S paveiksle panašumo sąlygos gauname:

$$\frac{h_{ct}}{h_{cr} - d_1} = \frac{\delta_{cr}}{w_{cr} - \delta_{cr}} = \frac{\epsilon_{ct,max}}{\epsilon_{s1} - \epsilon_{ct,max}}. \quad (S2.12)$$

Plyšio plotis išreiškiamas iš (12) lygties:

$$w_{cr} = \frac{\delta_{cr} (\epsilon_{s1} - \epsilon_{ct,max})}{\epsilon_{ct,max}} + \delta_{cr} = \frac{\delta_{cr} \Delta \epsilon_{s1}}{\epsilon_{ct,max}} + \delta_{cr}, \quad (S2.13)$$

čia δ_{cr} – plyšio viršūnės besiremianti į elemento neutraliąją ašį, pasiūlyta šio dydžio išraiška, įvertinant tiek plininę, tiek išorinę armatūrą.

Kadangi plyšio plotis gali būti susietas su elemento kreiviu per deformacijas, tai plyšio pločio kitimo stadijos bus analogiškos. Tokiu atveju bendroji plyšio pločio išraiška:

$$w_{cr} = w_{cr,0} - \Delta w_{cr,p} + \Delta w_{cr,y} + \Delta w_{cr,u} = w_{cr,0} + \frac{\delta_{cr} (\Delta \varepsilon_{s1,y} + \Delta \varepsilon_{s1,u} - \Delta \varepsilon_p)}{\varepsilon_{ct,max}}. \quad (S2.14)$$

Kintamasis (S2.13) ir (S2.14) lygtyse yra deformacijų prieaugis, kuris buvo nustatytas įvertinant betono ir kompozito dalinai standžios jungties darbą, kaip tai pavaizduota 1S paveiksle.

3. Eksperimentiniai tyrimai ir sustiprintų sijų laikomosios galios, įlinkio ir plyšio pločio skaičiavimo modelių tikrinimas

Trečiajame skyriuje pateikti gelžbetoninių sijų, sustiprintų iš anksto įtemptomis CFRP juostomis, eksperimentinių tyrimų rezultatai ir jų palyginimas su pasiūlytais skaičiavimo modeliais. Išbandyta 14 vienodų sijų (visos betonuotos vieno betonavimo metu), tačiau skyrėsi CFRP kompozito išankstinio įtempimo lygiai. Taip pat pristatomas autoriaus sukurta CFRP juostų išankstinio įtempimo įrenginys.

Sijų bandymai prasidėjo 174 dienos po betonavimo, taip eliminuojant betono traūkumo deformacijų efektus ir imituojuojant realią situaciją – dažniausiai bus stiprinamos ne naujos konstrukcijos. Išbandytos dvi serijos sijų: BS serijos sijos buvo sustiprintos neveikiant išorinėms apkrovoms, o BSM serijos sijos – veikiant išorinei apkrovai, kuri buvo imituojuojama spyruoklėmis. Išbandytų sijų duomenys pateikti S3.1 paveiksle ir S3.1 lentelėje. Išankstiniam įtempimui pasiekti pagaminta išankstinio įtempimo įranga, Europos patento paraiškos Nr. EP21167213.4. Stiprinimo ir išankstinio įtempimo įtaka gerai atspindima padidėjusiu lenkimo momentu, atitinkančiu takumo įtempius plieninėje armatūroje (takumo momentas, S3.2 pav.).

S3.1 lentelė. Eksperimentinių sijų parametrai

Sija	Betono stipris, f_{cm} , MPa	Armatūros takumo įtempiai, $f_{s1,y}$, MPa	CFRP stipris, f_f , MPa	CFRP tamprumo modulis, E_f , GPa	Išankstinis įtempimas σ_p , MPa	Pradinis lenkimo momentas M_0 , kNm
BS-0	50,00	568,50	2627,61	170,06	0	0
BS-20					522,08	0
BS-22					568,57	0
BS-48					1259,01	0
BS-49					1287,07	0
BSM-0-1					0	30,67
BSM-0-2					0	31,03
BSM-17					447,82	32,86
BSM-18					482,12	29,74
BS-35					919,17	29,16
BSM-46					1212,24	29,97
BSM-56					1471,59	29,72

S3.1 pav. Eksperimentinių sijų geometriniai parametrai

S3.2 pav. Sustiprintų ir kontrolinių sijų takumo momentų skirtumas procentine išraiška:

a) BS serija; b) BSM serija

Kai sijos buvo sustiprintos be išankstinio įtempimo, tai takumo momentas padidėjo apie 20 %, tačiau, kai CFRP juostos buvo įtemptos iki 50 % savo laikomosios galios, tai takumo momentas padidėjo beveik 100 %. Panašūs rezultatai gauti ir su elemento deformacijomis.

Siekiant geriau suprasti betono-kompozito jungties elgseną, palygintos betono ir kompozito deformacijos tuose pačiuose pjūviuose. S3.3 paveiksle pateikiamos 2 tipinės BS serijos sijos, o S3.4 paveiksle – BSM serijos sijos su panašiu CFRP įtempimo lygiu.

S3.3 pav. CFRP ir betono deformacijų palyginimas BS serijos sijose

S3.4 pav. CFRP ir betono deformacijų palyginimas BSM serijos sijose

Nustatytas laipsniškas betono ir kompozito jungties irimas ir vėlesnis inkarų išsijungimas į darbą BS serijos sijose, ir iškarto pasireiškiantis inkarų išsijungimas BSM serijoje. Remiantis eksperimentiniais duomenimis siūlomame modelyje įvertintas betono ir kompozito jungties standumą mažinantis koeficientas.

Validuojant siūlomus skaičiavimo modelius analizei panaudotos papildomos 78 kitų autorių išbandytos sijos.

Laikomosios galios palyginimas pateiktas S3.5 paveiksle ir S3.2 lentelėje.

S3.5 pav. Išbandytų sustiprintų sijų laikomosios galios palyginimas

S3.2 lentelė. Išbandytų sustiprintų sijų laikomosios galios palyginimas

Statistinis parametras	$M_{u,exp} / M_{u,calc},$ $\psi_f = \psi_{f,B}$	$M_{u,exp} / M_{u,calc},$ $\psi_f = \psi_{f,T}$	$M_{u,exp} / M_{u,calc},$ $\psi_f = 1$
Vidurkis	1,12	1,06	0,89
Standartinis nuokrypis	0,15	0,14	0,14
Variacijos koeficientas, %	13,83	13,66	15,82
Koreliacijos koeficientas, R	0,97	0,97	0,96

Nors statistinis skirtumas tarp laikomosios galios vertinant dalinai standžią jungtį ir standžią jungtį nebuvo didelis, bet taškai iš pavojingos zonos priartėjo prie saugios zonos. Atskirose sijose, nevertinant betono ir kompozito jungties standumo ($\psi_f = 1$), buvo pasiekta tik šiek tiek didesnė kaip 50 % laikomosios galios apkrova. Tai yra kritiškai nesaugu. Rezultatų palyginimas rodo, kad abu siūlomi jungties dalinio standumo vertinimo būdai gali būti naudojami skaičiuojant laikomąją galią.

BS serijos įlinkių palyginimas pateiktas S3.6 paveiksle, o BSM – S3.7 paveiksle.

S3.6 pav. Sijų įlinkiai: a) BS-0; b) BS-20; c) BS-22; d) BS-48; e) BS-49

S3.7 pav. Sijų įlinkiai: a) BSM-0-1; b) BSM-0-2; c) BSM-17; d) BSM-18; e) BSM-35; f) BSM-46; g) BSM-56

Eksperimentinių ir skaičiuotinių grafikų atitikimas įrodo, kad betono ir kompozito jungtyje yra praslydimas ir jis gali būti nustatytas pakankamai tiksliai. Kol BS serijos sijos yra išlinkę nuo išankstinio įtempimo jėgos, tempiamojo sustandėjimo koeficiente laipsnis $m \approx 1,00$, vėlesnėse stadijose jis yra tarp $1,00 \leq m \leq 2,00$. BSM serijos sijų atveju $m \approx 1,00$.

Papildomai pateikiamas įlinkio rezultatų palyginimas, įvertinant kitų autorių bandinius, grafiko lūžio taškuose: S3.8 paveiksle – ties takumo įtempiais, S3.9 paveiksle – ties elemento laikomąja galia.

S3.8 pav. Įlinkiai, atitinkantys takumo momentą: a) skaičiuotinių ir eksperimentinių rezultatų priklausomybė; b) paklaidos grafikas

S3.9 pav. Įlinkiai, atitinkantys laikomąją galią: a) skaičiuotinių ir eksperimentinių rezultatų priklausomybė; b) paklaidos grafikas

Bendroju atveju nustatyta, kad laipsnis m turėtų būti imamas lygiu 2. Taip pat nustatyta, kad net maži FRP kompozito išankstinio įtempimo lygiai įtraukia į darbą FRP kompozitus ir sumažina sustiprinto gelžbetoninio elemento įlinkį.

S3.10 pav. Lenkimo momento-plyšio pločio priklausomybės grafikai: a) BS-22; b) BS-20; c) BS-49; d) BS-48; e) BS-0; f) paklaidos grafikas

BS serijos lenkimo momento-plyšio pločio priklausomybės grafikai pateikti S3.10 paveiksle, BSM – S3.11 paveiksle, papildomų – S3.12 paveiksle.

S3.11 pav. Lenkimo momento-plyšio pločio priklausomybės grafikai: a) BSM-0-1; b) BSM-0-2; c) BSM-17; d) BSM-18; e) BSM-35; f) BSM-46; g) BSM-56; h) paklaidos grafikas

S3.12 pav. Kitų autorių išbandytų sijų lenkimo momento-plyšio pločio priklausomybės grafikai:
a) B3 ir B4 (Slaitas *et al.* 2018, Daugevičius *et al.* 2012, Daugevičius and Valivonis, 2013);
b) GCB-1; c) GCB-2; d) GCB-3 (Xing *et al.* 2018)

Lenkimo momento-plyšio pločio priklausomybės eksperimentinių ir skaičiuotinių grafikų palyginimas patvirtino siūlomą skaičiavimo modelį, taip pat patvirtino teiginį, kad

plyšio plotis gali būti apskaičiuotas pakankamai tiksliai ir plieninei armatūrai pasiekus takumo įtempius.

Bendrosios išvados

1. Šiuo metu statybos pramonėje naudojamoje FRP kompozito juostų išankstinio įtempimo įrangoje inkaravimo plokštelių paviršius yra lygus, todėl išankstinio įtempimo metu atsiranda kompozito juostos praslydimo rizika. Darbe pasiūlytoje išankstinio įtempimo įrangoje panaudotos profiliuotos su spygliais inkaravimo plokštelės, kurios patikimai inkaruoja FRP kompozito juostas. Tarp inkaravimo plokštelių ir kompozito juostų numatomas papildomas klijų sluoksnis neleidžia pažeisti CFRP struktūros.
2. Betono ir kompozito jungties irimas tarp dviejų plyšių išskiriamas kaip dažniausiai pasitaikantis kompozitu sustiprintų gelžbetoninių elementų suirimo atvejis. Todėl projektavimo normose yra siūloma naudoti skaičiavimo metodus, įvertinančius tokį suirimo atvejį. Tačiau literatūroje pateikiamuose modeliuose skaičiavimai remiasi irimo energija / irimo kriterijais ar ribiniais įtempiais išvestais iš mažos parametrinės apimties kompozito atplėšimo bandymų ir dažniausiai tinka tik išorinio klijavimo arba tik klijavimo į griovelius stiprinimo atvejams. Kai kurių tyrimų rezultatų analizė parodė, kad skaičiavimo modeliai, išskiriami kaip tiksliausi, turi pakankamai dideles paklaidas keičiant bandinių parametrus.
3. Betono ir CFRP kompozito jungties dalinis standumas gali būti vertinamas sudėtinių strypų teorijos pagrindu apskaičiuojamu koeficientu. Tačiau šio koeficiento apskaičiuojama reikšmė labai priklauso nuo jungties efektyviojo šlyties modulio nustatymo tikslumo. Todėl darbe taip pat siūlomas jungties efektyviojo šlyties modulio analitinis sprendinys ir kompozito ašinį standumą mažinantys koeficientai, kuriuos galima taikyti vertinant jungties irimą tiek tarp normalinių plyšių, tiek kompozito inkaravimo zonoje.
4. Eksperimentiniai tyrimai parodė, kad daugumoje BS serijos sijų užfiksuotas laipsniškas betono ir CFRP kompozito jungties irimas su palaipsniu inkarų įsijungimu į darbą, o BSM serijos sijose inkarai į darbą įsijungė iš karto. Šis reiškinys yra betono ir kompozito deformacijų skirtumo, kuris gali būti įvertintas pasiūlytu jungties standumo mažinimo koeficientu, priežastis.
5. Eksperimentinių ir skaičiuotinių rezultatų palyginamoji analizė atskleidė, kad elemento laikomoji galia su maža sisteminė paklaida (1,06–1,12), maža atsitiktinė paklaida (0,14–0,15), mažu variacijos koeficientu (13,66–13,83 %) ir dideliu koreliacijos koeficientu (0,97) buvo nustatyta pakankamai tiksliai. Todėl pasiūlytas skaičiavimo modelis gali būti naudojamas sustiprintų gelžbetoninių konstrukcijų laikomosios galios skaičiavimui. Nors statistinis skirtumas tarp laikomosios galios apskaičiuotos su sumažintu ir nesumažintu betono ir kompozito jungties standumo koeficientu skyrėsi neženkliai, tačiau laikomosios galios reikšmės iš pavoingos zonos persikėlė į saugumo pusę. Kai jungties standumas nebuvo mažinamas ($\psi_f = 1$) atskirų sijų eksperimentinė laikomoji galia pasiekė tik 50 % apskaičiuotos laikomosios galios.

6. Remiantis modifikuotu kietųjų kūrų irimo mechanika paremtu plyšio modeliu, pasiūlytas tiesioginis ryšys tarp betono plyšio parametrų ir deformacijų. Pasiūlytas modelis apima išankstinio įtempimo ir jungties dalinio standumo efektus. Skaičiuotiniai lenkimo momento-plyšio pločio priklausomybės grafikai pakankamai tiksliai atkartoja eksperimentinius taškus (koreliacijos koeficientas $0,95 \div 1,00$). Siūlomo skaičiavimo modelio validavimas kitų autorių tyrimų rezultatais parodė, kad plyšio plotis gali būti apskaičiuotas pakankamai tiksliai, netgi po to, kai tempiamoje plieninėje armatūroje pasiekiami takumo įtempiai.
7. Siūloma įlinkių skaičiavimo metodika paremta supleišėjusių ir nesupleišėjusių skerspjūvių modeliavimu nuosekliai sujungtomis spyruoklėmis, įtraukiant išankstinio įtempimo ir jungties dalinio standumo efektus. Eksperimentinių ir skaičiuotinių lenkimo momentų-įlinkių priklausomybės grafikų atitikimas ($R = 0,99-1,00$) patvirtina, kad betono ir FRP kompozito jungtis yra dalinai standi ir dalinio standumo efektas gali būti pakankamai tiksliai įvertintas siūlomu modeliu.
8. Nustatyta, kad netgi mažas kompozito išankstinis įtempimas jį įjungia į bendrą darbą su stiprinamąja gelžbetonine konstrukcija ir žymiai padidina sustiprintos konstrukcijos standumą. Jeigu FRP kompozitu stiprinami išorine apkrova neveikiami gelžbetoniniai elementai, kompozito iš anksto neįtempiant, stiprinimo efektas, iki plieninei armatūrai pasiekiant takumo įtempius, įlinkiui įtakos neturi.

Annexes¹

Annex A. Declaration of Academic Integrity

Annex B. The Co-Authors Agreements to Present Publications
Material in the Dissertation

Annex C. Copies of Scientific Publications by the Author on the
Topic of the Dissertation

¹The annexes are supplied in the enclosed compact disc.

Justas ŠLAITAS

FLEXURAL BEHAVIOUR OF REINFORCED
CONCRETE MEMBERS STRENGTHENED
WITH PRESTRESSED CFRP LAMINATE

Doctoral Dissertation

Technological Sciences,
Civil Engineering (T 002)

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