ANALYSIS OF METHODS FOR EVALUATION OF SOIL SHEAR STRENGTH PARAMETERS

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Abstract. Having performed an analysis of the methods for identifying a soil shear strength one can find four different coordinate systems for evaluating the soil shear strength parameters. It is stated that a theoretical functional relation exist for the shear strength parameters, have been identified via different evaluation methods. Thus one must obtain the same final parameters of shear strength. The investigation is assigned to identify reasons of obtained differing magnitudes of the shear strength parameters via triaxial testing by employing the used in practice methods. The method to identify angle of internal friction and cohesion satisfying all four coordinate systems is proposed.

Keywords: soil shear strength parameters evaluation methods, triaxial testing, angle of internal friction, cohesion.

1. Introduction

A determining the soil shear strength via direct shear apparatus. At his case the tangent of internal friction angle \( \tan \phi \) and the cohesion \( c \) are calculated according to normal and shear stresses acting at the failure plane having directly measured the acting forces (Aysen 2005; Craig 2004).

The perpendicular to sample surfaces stresses are measured when shear strength is investigated via triaxial testing (CНиП 2.02.02-85). The stresses at failure plane are calculated according to the above mentioned plane stresses having employed some assumptions. One of them states that the full stresses at the sample surfaces are the normal, id est they are principal ones. Thus, no shear stresses exist on the sample surface. Another assumption states that the sample collapses at plane inclined by the angle \( \phi + \theta \) to the minor principal stress. This angle is unknown value when processing the testing. Therefore one faces difficulties to determine normal and shear stresses at failure plane. Several methods are applied to solve the above mentioned problem (Parry 2004). The essence of the methods are as follow: the parameters of linear relationship are calculated for different coordinate systems, then employing theoretical relationships the magnitudes of the angle of internal friction and the cohesion are obtained.

The methods are valid also for the case when functional relationship exists for normal versus shear stresses at failure plane. When processing the triaxial testing data for more than 2 tests and when the sample strength parameters have scatter versus linear relationship, one obtains the different magnitudes of the angle of internal friction and the cohesion.

The authors calculated shear strength parameters by common methods and performed comparative analysis, identified the reasons, resulting the different internal friction angle and cohesion magnitudes. The proposed method for soil analysis via triaxial testing and that of processing of the data allows to evaluate each sample failure plane position for identifying the shear strength parameters.

2. Shear strength parameters evaluation methods

2.1 \( \sigma_1-\sigma_3 \) coordinate system

When the soil testing results are presented in \( \sigma_1-\sigma_3 \) coordinate system the angle of internal friction and the cohesion are obtained by following the certain procedures. First, the parameters \( \tan \theta \) and \( c_0 \) of linear relation \( \sigma_1 = f(\sigma_3) \) are obtained (Bukhartsev 1988) (Fig 1).

Fig 1. State of stresses via coordinates \( \sigma_1-\sigma_3 \)
Taking into account a linear relationship of the major \( \sigma_1 \) and minor \( \sigma_3 \) principal stresses (Amšiejus et al. 2009; Dirgelienė 2007; ČHN II 2.02.02-85), reading one obtains the magnitudes of the tangent of internal friction angle \( \tan \phi \) and the cohesion \( c \):

\[
\sigma_{1u} = \sigma_3 \cdot \tan^2 \left(45 + \frac{\varphi}{2}\right) + 2 \cdot c \cdot \tan \left(45 + \frac{\varphi}{2}\right),
\]

\[
\tan \phi = \frac{\tan \theta - 1}{2 \sqrt{\tan \theta}}, \tag{2}
\]

\[
c = \frac{c_0}{2 \sqrt{\tan \theta}}. \tag{3}
\]

### 2.2 q–p (Cambridge) coordinate system

When the soil testing results are presented in q–p coordinate system (Fig 2) (Parry 2004; Aysen 2005; Ranjan and Rao 2005), combining the mean principal stresses

\[
\frac{p}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_1 + 2 \sigma_3}{3}, \tag{4}
\]

and the deviatoric stresses

\[
q = \sigma_1 - \sigma_3, \tag{5}
\]

The \( q = f(p) \) parameters \( \tan \psi \) and \( c_\psi \) of the linear relationship are calculated by (Φ1959):

\[
c(\psi) = q - p \cdot \tan \psi. \tag{6}
\]

The angle of internal friction following the Mohr circle for a case \( c = 0 \) is described by:

\[
\sin \varphi = \frac{3 \tan \psi}{6 + \tan \psi}, \tag{7}
\]

and the cohesion:

\[
c = c(\psi) \cdot \frac{\sin \varphi}{\tan \psi}. \tag{8}
\]

### 2.3 t–s (Massachusetts) coordinate system

When the soil testing results are presented in t–s coordinate system (see Fig 3) (Craig 2004), combining the shear stresses

\[
t = \frac{\sigma_1 - \sigma_3}{2}, \tag{9}
\]

and the normal stresses

\[
s = \frac{\sigma_1 + \sigma_3}{2}, \tag{10}
\]

the parameters \( \tan \alpha \) and \( c_\alpha \) of the linear relationship

\( t = f(s) \) are obtained.

The Coulomb stresses envelope at failure is tangential to the Mohr stresses circle with a slope \( \varphi \) and the intercept \( c \) (Vervečkaite et al. 2007). The failure line for t–s coordinate system crosses a point X (see Fig 4). The line \( K_f \) has a slope \( \alpha \) and an intercept coordinate \( c_\alpha \).

The values of \( \alpha \) and \( c_\alpha \) by applying the t–s plot can be converted into \( \varphi \) and \( c \) by employing the equations (13) and (14):

\[
\varphi = \sin^{-1} (\tan \alpha), \tag{13}
\]

\[
c = \frac{c_\alpha}{\cos \varphi}. \tag{14}
\]
2.4 \( \tau - \sigma \) coordinate system

The major \( \sigma_1 \) and minor \( \sigma_3 \) principal stresses of the sample are measured directly during triaxial testing process. One faces the problem when identifying the failure angle of the plane in which the shear and the normal stresses \( \tau \) and \( \sigma \) act (Fig 5). Therefore the \( \tau - \sigma \) coordinate system is employed rather seldom.

The soil shear strength \( \tau_u \) at failure plane depends on the normal stresses \( \sigma \) acting on the failure plane:

\[
\tau_u = \tan \varphi \sigma + c .
\] (15)

The normal stress at the failure plane is described by:

\[
\sigma = (\sigma_1 - \sigma_3)\cos^2 \alpha + \sigma_3 .
\] (16)

The shear stress at the failure plane is described by:

\[
\tau = 0.5 \sin 2\alpha (\sigma_1 - \sigma_3) .
\] (17)

3. Evaluation of shear strength parameters via triaxial testing by employing different methods

The modified angle of internal friction and the cohesion have been calculated following the methods described in sections 2.1–2.3 (see Figs 7–9) by employing the triaxial testing experimental data. The above mentioned magnitudes were converted into \( \tan \varphi \) and \( c \) by using the equations 2–3, 7–8, 13–14. Having performed an analysis of the results one can find small variation of the angle of internal friction, but the cohesion varies significantly within bounds of 5 and 10 kPa (see Table 1).

Having processed the same experimental soil testing data by different common methods one finally obtains different magnitudes of the \( \tan \varphi \) and the \( c \). A question arises: which magnitudes are true? The validation can be performed as follows.

By employing the each common method for calculated the \( \tan \varphi \) and the \( c \) magnitudes one can identify the normal and shear stresses acting on the failure plane. Then applying the least squares method one can identify the magnitudes \( \tan \varphi^* \) and \( c^* \) according these magnitudes (Eqs 18–19). If they coincide with the ones, obtained by the common method, one can state the method to be applicable for processing of the shear strength parameters. But results presented in Table 1 obviously show the difference of \( \tan \varphi, c \) and \( \tan \varphi^*, c^* \) obtained via the different methods.
Table 1. Mean values of soil shear strength parameters \( \tan \varphi \) and \( c \) (in kPa) calculated by common and proposed methods

<table>
<thead>
<tr>
<th>Common methods</th>
<th>( \sigma_1-\sigma_3 ) coordinate system</th>
<th>q-p coordinate system</th>
<th>t-s coordinate system</th>
<th>Proposed method τ-σ coordinate system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified shear strength parameters</td>
<td>( \tan \theta = 4.127 ) ( c_\theta = 43.5 )</td>
<td>( \tan \psi = 1.553 ) ( c_\psi = 11.99 )</td>
<td>( \tan \alpha = 0.618 ) ( c_\alpha = 4.05 )</td>
<td>–</td>
</tr>
<tr>
<td>Modified shear strength parameters converted into ( \tan \varphi ) and ( c ) (according to formulae 2,3,7,8,13,14,18,19)</td>
<td>( \tan \varphi = 0.770 ) ( c = 10.71 )</td>
<td>( \tan \varphi = 0.784 ) ( c = 6.05 )</td>
<td>( \tan \varphi = 0.786 ) ( c = 5.16 )</td>
<td>( \tan \varphi = 0.780 ) ( c = 7.27 )</td>
</tr>
<tr>
<td>Shear strength parameters calculated according to ( \sigma ) and ( \tau ) acting in failure plane</td>
<td>( \tan \varphi^* = 0.780 ) ( c^* = 7.24 )</td>
<td>( \tan \varphi^* = 0.780 ) ( c^* = 7.28 )</td>
<td>( \tan \varphi^* = 0.780 ) ( c^* = 7.29 )</td>
<td>( \tan \varphi^* = 0.780 ) ( c^* = 7.27 )</td>
</tr>
</tbody>
</table>

4. Analysis of methods for evaluating shear strength parameters

When one employs the data only of the two tests the magnitudes of the values \( \tan \varphi \) and \( c \) being evaluated by the different methods coincide. The same result is obtained for larger number of tests if the functional relationship between principal values exists. But when processing the experiments with significant variation of results one obtains the different magnitudes of \( \tan \varphi \) and \( c \) when employing the different methods.

The variation of the shear strength parameters being identified by the different methods is influenced by the failure plane location \( \alpha = 45^\circ + \varphi/2 \), treated differently for separate methods. The different parameters are minimized in different coordinate systems, yielding the different failure planes, subsequently resulting the different magnitudes of the shear strength parameters.

At the standard \( \sigma_1-\sigma_3 \) coordinate system case one minimizes the differences squares sum \( \sum_{i=1}^{n} (\sigma_{i1} - \sigma_{i3})^2 \) of the major principal stress magnitudes \( \sigma_{i1} \) being identified experimentally and that of the soil shear strength \( \sigma_{i3} \) magnitudes being calculated by Eq (1).

At t-s coordinate system case one minimizes the differences squares sum \( \sum_{i=1}^{n} (t_i - t_{ui})^2 \) of the largest shear stress \( t_i = \sigma_{i1} - \sigma_{i3}/2 \) and that of the soil shear stress \( t_{ui} \) magnitudes being identified according to the Coulomb law \( t_{ui} = s_i \cdot \tan \alpha + c_\alpha \).

At q-p coordinate system case one minimizes \( \sum_{i=1}^{n} (q_i - q_{ui})^2 \), id est the differences of principal measured stresses \( q_i = \sigma_{i1} - \sigma_{i3} \) and that of the \( q_{ui} \) being identified according to the \( q_{ui} = \sigma_{i1} \cdot \tan \psi + c_\psi \).
5. Proposed method for identifying shear strength parameters due triaxial testing data

What one should minimize actually if the minimizing magnitudes differ for different coordinate systems?

The proposed method offers to minimize the differences sum $\sum_{i=1}^{n} (\tau_i - \tau_{ui})^2$ of the soil shear strengths individual magnitudes being identified experimentally and that of the soil shear strength magnitudes being calculated according to Coulomb law. Here one replaces $\tau_i$ by the magnitudes of shear stresses being identified by the Eq (17), and replaces $\tau_{ui}$ by the magnitudes identified by the Eq (15). $\sigma$ in the Eq (15) is replaced by the magnitudes of the normal stresses obtained by the Eq (16). The failure plane angle in respect of minor principal stress is calculated by $\alpha = 45^\circ + \varphi/2$. Thus, one obtains:

$$
\sum_{i=1}^{n} (\sigma_1 - \sigma_3)^2 \cdot (1 - \frac{\tan^2 \varphi}{\tan^2 \varphi + 1})/2 + 2 \cdot \sum_{i=1}^{n} (\sigma_1 - \sigma_3) \cdot \sum_{i=1}^{n} \sigma_3,
$$

$$
((1 - \frac{\tan^2 \varphi}{\tan^2 \varphi + 1})/2 - \sum_{i=1}^{n} \sigma_3)^2 \cdot (1 - \frac{\tan^2 \varphi}{\tan^2 \varphi + 1})/2 + 2 \cdot \sum_{i=1}^{n} (\sigma_1 - \sigma_3) \cdot \sum_{i=1}^{n} \sigma_3,
$$

$$
((1 - \frac{\tan^2 \varphi}{\tan^2 \varphi + 1})/2 - \sum_{i=1}^{n} \sigma_3)^2 \cdot (1 - \frac{\tan^2 \varphi}{\tan^2 \varphi + 1})/2 - 2 \cdot \sum_{i=1}^{n} (\sigma_1 - \sigma_3) \cdot \sum_{i=1}^{n} \sigma_3 \cdot ((1 - \frac{\tan^2 \varphi}{\tan^2 \varphi + 1})/2)
$$

Having solved the equation (21) in respect of the single unknown $\tan \varphi$, one obtains the $\tan \varphi = 0.780$ magnitude. Then $c = 7.27$ kPa is determined via equation (22) (see Table 2). The latter magnitudes correspond the magnitudes of $\tan \varphi$ and $c$, being obtained by minimizing the differences squares sum of the individual shear strength parameters and the magnitudes calculated according the Coulomb law. The latter magnitudes are different from magnitudes being obtained by the common methods (see Table 2).

The available arguments of $\tan \varphi$ magnitudes included the equation $f(\tan \varphi) = 0$ are presented in Fig 10. One can find from the graph that equation has a single solution.

![Fig 10. Graph of equation $f(\tan \varphi) = 0$](image_url)
6. Data of bearing resistance calculation

The characteristics values according to the requirements of EC 7 of converted shear strength parameters obtained by the common methods of processing of the triaxial testing data were calculated for coefficients of variation \( V_\sigma = 0.05 \) and \( V_\text{co} = 0.3 \) (Schneider 1999; Fellin et al. 2005; Amšiejus and Dirgėlienė 2007). The magnitudes of angle of internal friction were obtained to be similar when the cohesion varied from 2 kPa till 5 kPa (see Table 2).

The design bearing resistance \( R_d \) was calculated for drained conditions by means of the methods provided in standard documents EC 7. A spread foundation is loaded centrifically. The foundation width (see Table 2) was calculated according to the design approach 3 by employing the triaxial testing results. The magnitudes of the angle of internal friction were obtained to be 1.70 m and 1.75 m. Obviously foundation width varies more for the larger \( V_\sigma \) and \( V_\text{co} \).

Table 2. Results of calculation of design bearing resistance according EC7

<table>
<thead>
<tr>
<th>Methods</th>
<th>( \sigma_1-\sigma_3 ) coordinate system</th>
<th>( q-p ) coordinate system</th>
<th>t-s coordinate system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristics values of shear strength parameters ( \tan \varphi ) and ( c ), (in kPa)</td>
<td>( \tan \varphi = 0.705 )</td>
<td>( \tan \varphi = 0.719 )</td>
<td>( \tan \varphi = 0.721 )</td>
</tr>
<tr>
<td>( c_z = 5.4 )</td>
<td>( c_z = 3.1 )</td>
<td>( c_z = 2.6 )</td>
<td></td>
</tr>
<tr>
<td>Foundation width magnitude ( B ), (in m)</td>
<td>1.70</td>
<td>1.74</td>
<td>1.75</td>
</tr>
</tbody>
</table>

The characteristics values of the \( \tan \varphi = 0.716 \), \( c_z = 3.7 \) kPa and subsequently the foundation width \( b = 1.73 \) m were calculated for the shear strength parameters evaluated by proposed method for variation coefficients \( V_\sigma = 0.05 \) and \( V_\text{co} = 0.30 \).

Conclusions

1. Currently the shear strength parameters identified by employing the triaxial testing are evaluated by the different methods. The magnitudes of the angle of internal friction and the cohesion are obtained differently for the same primary testing data. The internal friction angle magnitude varies insignificantly, when comparative difference of the cohesion reaches 51 \%

2. The shear strength parameters being processed via the different methods do not coincide as the different magnitudes are minimized for identifying their mean values by the least squares method, e.g. differences squares sum of the largest principal stresses or the principal stresses from their calculated magnitudes, being calculated by regression equation.

3. The mean shear strength parameters obtained by the proposed method are calculated by minimizing the differences squares sum of the individual shear strengths at the failure plane and that of being calculated by the regression equation.

References


REFERENCES


