Abstract. This paper briefly reviews the load-carrying capacity predictive models for dowel-type connectors in timber to timber connections as well as composite timber to concrete connections. The short analyses based on existing theoretical and experimental investigations of researchers showed that the connections with inclined screws possess the greater load-carrying capacity and stiffness as those with perpendicular to the shear plane arranged fasteners. Analyses of existing predictive theoretical models represented by numerous authors have showed that there is no computational model for connections with inclined screws and in practice only the empirical formulas and only for particular screw type may be used. Recently the calculation methods for dowel type connections as well as for screws are based on Johansen’s (1949) theory. This model lets to determine the load carrying-capacity very well for bolted or nailed connections, and only satisfactorily for screwed connections. Another theoretical model of Коченов (1953) which was adopted in timber design codes in Russia is based on the limited value of deformation of compressed under the fastener timber grain, and enables to relate withdrawal deformation with embedding deformation. In this paper the theory of Коченов was extended and applied to timber to concrete connections with inclined screws; the possible failure modes are described, equations for load-carrying capacity derived, and the main characteristic parameters for connection are determined. The theoretical model assessed with experimental results also carried-out by other researchers.

Keywords: timber-concrete composite, inclined screws, load-carrying capacity, embedding and withdrawal capacity.

Introduction

Timber-concrete composite systems in nowadays are widely used technique in repair and reconstruction of old dwelling and especially being possessed of historical heritage buildings. By applying the concrete layer on a system of timber beams and connecting those by mechanical fasteners the achieved accessible system is capable to resist existing and additional actions. This technique lets us to save the old original structural timber elements and improve the properties of ceilings such as sound and fire resistance; to improve the load-carrying capacity and prepare it for further exploitation.

The screws as connectors are used widely for timber to concrete connections due to its popularity in timber to timber connections and easy installation. In a traditional way they are arranged perpendicular to the timber grain (and also to shear plain) but in this way the effective stiffness of connection could not be reached. The arrangement of screws perpendicular to the shear plane provides high slenderness for the connection, because of the bending actions on fastener and large embedding deformations of the timber grain.

Although the inclined screws in timber-concrete composite beams in practice are used increasingly and the shape of the screws and of the threads are refining the formulas for design of load-carrying capacity for connections are not set.

This paper represents the analytical load-carrying capacity predicting model for timber-concrete composite connections with inclined screws; analyses the possible failure modes, presents the derived formulas, and influencing parameters; produces the comparison between the analytical solutions and experimental tests results given also by other researchers.

Load-carrying capacity of screwed connections

Recently the methods of calculation of load-carrying capacity of dowel type connections are based on Johansen’s (1949) theory, which assumes the behaviour of connected members as well as fastener material as rigid-plastic. This assumption lets to determine the load
The analytical model for timber to concrete connections with inclined screws

Kochnov (1953) theory can be applied to timber to concrete connection with inclined screws because of possibility to relate the embedding and withdrawal deformations of the timber. This theoretical model which was adopted in timber design codes in Russia is based on more realistic behaviour of element material and on the limited value of embedding deformations \( \delta_{h,el} = 2 \delta_{h,pl} \) of the timber under the fastener. The basic scheme for fastener loaded by moment and shear forces as well as stress distribution in timber is shown in Fig 1.

![Fig 1. The basic scheme for fastener; distribution of stresses and deformations](image)

Using this scheme (Fig 1) and changing the values for \( \beta_H \) (Eq. 1) it is possible to obtain the load-carrying capacity for the fastener in the range from the elastic till the plastic solution. Parameter \( \beta_H = h_2/h_1 \) – is the ratio of the lengths of elastically and plastically compressed timber grain under the screw, and accordingly to Fig 1 can be expressed so:

\[
\beta_H = \frac{1}{\frac{\delta_h}{\delta_{h,el}} - 1}
\]  

Equation (1)

According to Kochnov (1953) it is possible to obtain the load-carrying capacity from elastic to the plastic solutions changing the values of parameter \( \beta_H \) from 0 to \( \beta \), though at the marginal cases (elastic and plastic) the solution is a little inaccurate because of the accepted scheme (Fig 1). Assuming the inclination angle of the fastener is 90° and changing the ultimate embedding deformations thereby the value of parameter \( \beta_H \) the obtained relative load-carrying capacity per fastener \( T/T_{ad f_h} \) is graphically represented in Fig 2, where relative eccentricity \( m \) is:
where: \( e_j \) – eccentricity of the joint; \( M_j \) – moment in the fastener at the point of shear plain; \( T \) – in the fastener at shear plain acting shear force; \( a \) – member thickness or embedding length of the fastener.

The bending moment increases, thereby and eccentricity – the longer fastener in timber member under which the timber grain is compressed and deformed plastically. The maximum value for eccentricity \( m \) is 0.5 (Fig 2) – it means that timber grain yields plastically under the fastener at full length, and the maximum value of relative load-carrying capacity is reached.

![Fig 2. Relationship between \( \bar{T} \) and \( m \) for single shear connections](image)

As can be seen from Fig 2 the load-carrying capacity of single shear timber to steel (for thin plate, a bending moment \( M_j = 0 \) ) connection is equal to \( 0.36 f_h da \) (when \( \delta_h / \delta_d = 2 \)) and \( 0.414 f_h da \) (when \( \delta_h / \delta_d \approx 4.4 \)) according to Коченов and Johansen respectively. This observation shows that Коченов model is more universal.

According to the results of experimental tests (Kavaliauskas 2010) the simplified linear (bilinear) relationship between the stresses and slip deformations for withdrawal and embedding strength of timber was assumed (Fig 3).

The following main assumptions for predictive load-carrying capacity model have been accepted:

- The behaviour of timber under the screw is simplified to the bilinear elastic plastic and to the linear elastic for embedding and withdrawal strength respectively (Fig 3):

\[
\begin{align*}
\sigma_h &= \begin{cases} 
\sigma_{h,\alpha} & \text{for embedding}, \\
\sigma_{h,\alpha} & \text{for withdrawal}.
\end{cases}
\end{align*}
\]

- The bending capacity of the screw is taken as yield moment \( M_j \);
- The fixing of the screw in concrete is assumed as a stiff;
- The failure modes are defined at a limit value of deformations: \( \delta_{h,u} = 2 \delta_{h,el} \) – for embedding, and \( \delta_{ax,u} = \delta_{ax,el} \) – for withdrawal deformations;
- The equations of equilibrium are written on a non-deformed screw axis;
- The elongation of the screw due to the tensile forces is neglected.

Changing the angle between the screw axis and the timber grain also changes the value of resultant embedding and withdrawal stresses in timber grain under the screw (Fig 4).

![Fig 3. The real and the idealized curves for embedding (1) and withdrawal (2) stress-deformations relationships](image)

![Fig 4. Distribution of stresses under the screw shank](image)

When the angle \( \alpha \) is 90° only the embedding stresses \( \sigma_h \) direction of which is identical with timber grain act, and the withdrawal stresses \( \sigma_{ax,\alpha} \) are equal to 0. Decreasing the angle value from 90° to 0° the resultant of embedding stresses approaches to zero and withdrawal – to the value of \( T/\alpha \) (where: \( a \) is the embedding length of the screw shank in the timber; \( d \) – the outer diameter of embedded screw shank). The equilibrium of internal forces according to Fig 4 is:

\[
T = ad(\sigma_h \sin \alpha + \sigma_{ax,\alpha} \cos \alpha)
\]
The greater decreases of angle $\alpha$ – the greater increase of the withdrawal part of a whole slip deformation of the screw, which determines the load-carrying capacity of the screw.

Because of the different behaviour of embedded and withdrawal timber under the screw at the ultimate case of connection (failure mode) the values of embedding and withdrawal stresses are not of the ultimate values. Therefore, the Eq.3 may be rewritten so:

$$ T = ad\left(\beta_h f_h \sin \alpha + \beta_{ax} f_{ax,\alpha} \cos \alpha\right) $$

(4)

where: $\beta_h = \sigma_h / f_h$ and $\beta_{ax} = \sigma_{ax} / f_{ax}$ – are the parameters, showing the intensity of embedding and withdrawal stresses respectively at the failure mode of connection, and are the functions depending on the angle $\alpha$ and deformations $\delta_{ax}$ and $\delta_h = f(\alpha, \delta_{ax}, \delta_h)$.

The characteristic three possible failure modes with the internal forces and with distribution of stresses as well as deformations for screwed connection are shown in Fig 5.

![Fig 5. First (a) and second (b) failure modes: stress distribution and deformations](image)

The first failure mode is characterised by the ultimate embedding $\delta_{h,u} = 2\delta_{h,el}$ or withdrawal $\delta_{ax,u} = \delta_{ax,el}$ deformations. The relations between the embedding and withdrawal deformations for the first failure mode are:

$$ \delta_h / \delta_{ax} = \tan \alpha $$

(5)

$$ \delta_{h,u} = 2\delta_{h,el} = 2f_h / C_h $$

(6)

$$ \delta_{ax,u} = \delta_{ax,el} = f_{ax,\alpha} / C_{ax,\alpha} $$

(7)

where: $C_h = f_h / \delta_{h,el}$ and $C_{ax} = f_{ax} / \delta_{ax,el}$ – the elastic deformability parameters for embedding and withdrawal.

For simplification of load-carrying capacity equations the following parameters in equilibrium equations were used: $k_c = C_h / C_{ax,\alpha}$, $k_{f,\alpha} = f_h / f_{ax,\alpha}$ and assuming that values of $C_{ax,\alpha} > C_h$ according to Fig 3 it is possible to distinguish two values for the angle $\alpha - \alpha_h$ and $\alpha_{ax}$ from the following:

$$ \tan \alpha_h = \frac{\delta_{h,u}}{\delta_{ax,u}} = \frac{2f_h}{k_c} = 2k_f $$

(8)

$$ \tan \alpha_{ax} = \frac{\delta_{ax,el}}{\delta_{ax,u}} = \frac{f_h}{k_c} = \frac{k_f}{k_c} $$

(9)

where: $k_f = f_h / k_c$.

Then the relative load-carrying capacity $\overline{T}$ equations for the first failure mode will be:

$$ \overline{T} = \begin{cases} \sin \alpha + \frac{2k_f}{k_c} \cos \alpha, & \alpha \geq \alpha_h \\ \sin \alpha + \frac{1}{k_{f,\alpha}} \cos \alpha, & \alpha_{ax} \leq \alpha \leq \alpha_h \\ \tan \alpha + \frac{1}{k_{f,\alpha}} \cos \alpha, & \alpha \leq \alpha_{ax} \end{cases} $$

(10)

The first failure mode is appropriate for relatively short screws, when at ultimate state the timber grain yields plastically along the screw shank.

Increasing the length of the screw, the embedding stresses distribute non-uniformly (Fig 5) and the induced bending moment at shear plain can reach an ultimate value $- M_s$. The resultant relative embedding force (in some cases and the relative load-carrying capacity) can be found from the equation of equilibrium of bending moments:

$$ h^3 - 3a^2 \beta_H \left(1 + \beta_H - 2\mu\right)h_3 + 2a^3 = 0 $$

(11)

$$ \beta_h = \left[1 + \beta_H - \frac{1}{2}\beta_H \left(\frac{h_3}{a} - \frac{a}{h_3}\right)\right] $$

(12)

The parameter $\mu$ defines the ratio between the ultimate bending moment of the screw and the superficial bending moment induced by plastically yielded timber along whole embedded screw’s length $a$:

$$ \mu = \frac{\pi a^2 f_h}{32 a^2 / h_3} $$

(13)

From the Eq. (12) the plastically deformed length of timber grain under the screw $h_3$ can be obtained, and herewith the load-carrying capacity:

$$ \frac{T}{ad f_h} = \beta_h \sin \alpha + \frac{\beta_{ax}}{k_{f,\alpha}} \cos \alpha $$

(14)
Assuming the ultimate value for embedding deformation $\delta_h$ according to the Fig 5 the screw’s turn angle $\theta$ at failure mode can be found:

$$\frac{\delta_h}{\delta_{h,el}} = 1 + \beta_H$$

$$\delta_l = \delta_{h,el} \frac{h_1}{h_2}$$

$$\tan \theta = \frac{\delta_h + \delta_l}{a}$$

Denoting the distance from the shear plain to the point where the embedding stresses are equal to 0 as $a_0$ the values for withdrawal and slip deformations of connection can by found:

$$a_0 = h_2 + h_3 = h_1(1 + \beta_H)$$

$$\delta_{ax} = a_0 \left( \frac{\sin \alpha}{\sin(\alpha - \theta)} - 1 \right)$$

$$\delta = \delta_h \sin \alpha + \delta_{ax} \cos(\alpha - \theta)$$

As in the case of the first failure mode the value of parameter $\beta_{ax}$ is equal to $\beta_{ax} = \delta_{ax}/\delta_{ax,el} \leq 1$. When the direct relationship between the embedding and the withdrawal deformation absent it is not possible to determine the load carrying capacity directly – so only the approach way is possible: first assuming the minimum value for parameter $\beta_H = 1$ and solving equations (11), (12) and (15–20) the value of withdrawal deformation can be obtained. If that value is lower than the ultimate one ($\delta_{ax} < \delta_{ax,el}$) the load-carrying capacity of the screw can be obtained by Eq. (14); if the value of $\delta_{ax} > \delta_{ax,el}$, the value of $\beta_H$ should be increased and calculations repeated. For approximate calculations the relationship between the values of embedding and withdrawal deformations can be assumed as for the first failure mode Eq. (5). So the equations for relative load-carrying capacity for second failure mode are:

$$\bar{T} = \begin{cases} 
\beta_h \sin \alpha + \frac{\beta_{ax}}{k_{f,\alpha}} \cos \alpha, & \text{when } \alpha \geq \alpha_h \\
\beta_h \sin \alpha + \frac{1}{k_{f,\alpha}} \cos \alpha, & \text{when } \alpha_{ax} \leq \alpha \leq \alpha_h \\
\frac{\beta_h \tan \alpha}{k_{f,\alpha}} \sin \alpha + \frac{1}{k_{f,\alpha}} \cos \alpha, & \text{when } \alpha \leq \alpha_{ax}
\end{cases}$$

By increasing the embedding length of the screw in timber, the bending moment increases herewith, and the third failure mode occur when two plastic hinges in the screw develop.
The third failure mode is appropriate for the relatively long dowel type screws, in this case screws. This failure mode is identical in both Johansen’s (1949) and Kepes (1951) theories. So the length of plastically deformed timber grain under the screw $h_3$ and herewith the relative load-carrying capacity $\bar{T}$ may be obtained from the equation of equilibrium of bending moments (Fig 6):

$$M_j = T_{ax} - 0.5 T_{ax} + M_j$$

$$\bar{T} = \frac{h_3}{a} = \frac{a_h}{a} = 0.627 \frac{d}{a} f_y$$

The value of parameter $\mu$ dividing the second and third failure modes may be found in approach way equating the formulas (12) and (23) with known value of $\beta_H$; assuming $\beta_H = 1$ the result for $\mu \approx 0.044$.

The parameter $\mu$ depends on the value of bending capacity of the screw $M_j$, herewith on nominal value of tensile strength of the steel. In this case the value was obtained elastic section modulus $W = \pi d^3/32$ multiplied by tensile strength value of steel including strain hardening $f_y = 0.8 f_y$ (Scheer et al. 1988):

$$M_j \approx 0.079 \cdot f_y \cdot d^3$$

The equations for load-carrying capacity of the screws inclined at any angle $\alpha$ in respect to the shear plain may be found as for the second failure mode from the Eq. (21) with parameter value $\beta_h$ from Eq. (23).

The shown curves (Fig 7) have three parts: left-, middle- and right-side. The left-side is elastic solution of the load-carrying capacity of the screw and may be obtained treating the screw as beam on two directional elastic foundations (longitudinal and transverse). The middle-side is elastic-plastic solution and can be solved by approach way (mentioned above). The right-side part of the curves is the exact solution of developed model.

It is evident that the maximum load-carrying capacity of the relatively short screws is at values of an angle $\alpha = \alpha_{ax}$ – approximately $50^\circ$ and this value corresponds to the elastic solution. The exact solution when $\alpha < \alpha_h$ found for second and third failure modes (Fig 7) is depicted by dots. The large deviation from exact solution is 13% at the point between elastic and elastic plastic parts.

The parameter $\beta_h$ is the minimum value of two ones obtained from Eq. (12) and Eq. (23).

When the inclination angle of the screw is $90^\circ$, second member in the first formula of Eq. (28) turns into zero, though the withdrawal capacity has significant increase in load-carrying capacity also for this arrangement. In such case the withdrawal capacity can be evaluated calculating the angle $\theta$ value at the failure mode, and the first member in Eq. (28) with second one:

$$T = \left( \beta_h + \frac{\cos \left( 90^\circ \theta \right)}{k_{\beta,90}} \right) a f_h$$

Certainly, the value for turn angle $\theta$ of screw shank in timber member at failure mode may be obtained only with known values of deformability parameters $C_h$ and $C_{ax}$. For these parameters there are no formulas derived so they should be determined by the tests, and depends on shape of thread of screw and angle between timber grain and load (normally and screw longitudinal axis).
Evaluation of the developed model

By the tests (Kavaliauskas et al. 2007) obtained mean value of load-carrying capacity of connection with screws inclined at an angle $45^\circ$ is $F = 4669 \text{kN}$, and with here represented theoretical model (not simplified) computed one with second member of Eq. (28) is $T = 4341 \text{kN}$ consequently the ratio – $F/T = 1.07$. The theoretically and experimentally obtained failure mode of the screw is characterised by one plastic hinge at shear plain and exhausted withdrawal capacity coincide (Kavaliauskas 2010) (Fig 8).

![Fig 8. Composite timber-concrete connection after test (Kavaliauskas et al. 2007)](image)

In Fig 8 on the left side is shown the threaded part of the screw imbedded in timber member and deformed at failure mode, and on the right side – not collapsed the concrete prism of specimen.

While most of researchers have experimentally determined the stiffness and load-carrying capacity of connections, but the withdrawal and embedding capacities of timber as well as the tensile strength of used screws were not obtained, although these characteristics are very important. Therefore, evaluating the theoretical model these parameters were calculated in accordance with Eurocode 5 (2004) formulas and in accordance with researchers given values of timber density of. This may led to the results of uncertainty. The experimentally obtained and theoretically computed values of load-carrying capacities of screws are compared in Fig 9. In ordinate and absissa axis are depicted theoretical and experimental values respectively (in kN).

![Fig 9. Comparison between the theoretical ($T$) and experimental ($F$) values of load-carrying capacity of composite timber-concrete connections](image)

The mean values of ratios $F/T$ compared the results found in literature (Blass 1999, Faust and Selle 1999) when inclination angles are $45^\circ$ and Künng’s (1987) when screws are inclined at angle of $60^\circ$ is equal to 1.05 and 1.17 respectively.

Conclusions

The theoretical model was developed on the base of Кочо́в (1953) model by applying the additional equations for the embedding and withdrawal deformation of timber grain under the screw. The developed model allows calculating the load-carrying capacity as well as the corresponding deformation, and precisely evaluates the failure mode of connections.

The parameter $\mu$ was involved which shows the number of possible to occur plastic hinges in the screw at the failure mode of connection. The characteristic values for this parameter were defined, and they are equal to – 0.5 and 0.044. When $\mu > 0.5$, the plastic hinge in the screw does not develop – the first failure mode. When $0.5 \geq \mu > 0.044$ one plastic hinge develops in the screw – the second failure mode; and when $\mu < 0.044$ – two plastic hinges develop in the screw – the third failure mode.

The values of characteristic angles for connections with inclined screws $\alpha_h$ and $\alpha_{ax}$ between the screw axis and the timber grain in association with parameters $\mu$ and $k_c$ were distinguished. By the values of $\alpha_h$ and $\alpha_{ax}$ the ductility of connection can be supposed. For relatively short screws ($\mu \geq 0.5$) the effective inclination angle is near the value of 50°; the longer the screw the value of angle is closer to the 45°, and for very long screws this value is close to zero. Therefore, the load-carrying capacity is more often determined with tensile strength of screw. Embedding and withdrawal capacity of timber is fully used when $\alpha = \alpha_h$ and not always represents the maximum of load-carrying capacity. This value ($\alpha_h$) is particularly depended on value of parameter $k_c$.

The simplified computational model for the connection load-carrying capacity was proposed. It is based on the ultimate embedding deformation $2\delta_{h,el}$ of the timber grain and with the value of deformation factor $k_c = 0.4$, but without the values of parameters $C_h$ and $C_{ax}$, so the slip deformation can’t be calculated.

The theoretical and experimental results were compared – disagreement was only 7%. The experimental results of others researchers were compared with the theoretically obtained values by the developed simplified computational model. The mean values of the scatter of the results equal to 1.05 and 1.17 for angles of inclination $\alpha = 45^\circ$ and $\alpha = 60^\circ$ respectively. The result shows that model is capable to predict respectably the load carrying capacity of timber-concrete composite connection with at any angle in respect to shear plain inclined screws.
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