RELIABILITY INDEX DESIGN IN REINFORCED CONCRETE STRUCTURES OF ANNULAR CROSS SECTIONS

Antanas Kudzys¹, Romualdas Kliukas²

¹Kaunas University of Technology, Tunelio g. 60, 44405 Kaunas, Lithuania. E-mail: asi@asi.lt
²Vilnius Gediminas Technical University, Saulėtekio al. 11, 10223 Vilnius, Lithuania. E-mail: pirmininkas@vgtu.lt

Abstract. An application of concrete members of annular cross sections reinforced by steel bars uniformly distributed throughout their perimeters to building columns, bridge piers, carriers of overhead power transmission lines and their design methods are discussed. The modelling of resisting compressive forces and bending moments of eccentrically loaded cross sections as particular members of tubular members and their statistical parameters are presented. The second order effects of columns, piers and poles caused by their structural deformations are analysed. The time-dependent margins of particular members exposed to extreme loading situations due to floor and climate extraordinary actions are considered. The instantaneous survival probability of particular members and its application features in probability-based design of columns, piers and poles are developed. The long-term survival probability and reliability index design of particular and structural members are based on the unsophisticated method of transformed conditional probabilities. The analysis of revised reliability indices for structures subjected to fixed values of abnormal extraordinary actions is based on Bayesian approach.

Keywords: columns, piers, poles, annular sections, combination of actions, second order effects, survival probability, reliability index, Bayesian approach.

1. Introduction

The economically and structurally effective reinforced concrete members of annular cross sections may be treated as reliable building columns, bridge piers and carriers for overhead power transmission lines. Their safety and durability depend basically on recurrent extreme loading situations that are likely to occur during the reference working life of buildings, bridges and transmission lines (Fig 1).

The dangerous action effects of building columns may be caused by extreme wind or snow loads. The members of decks and piers of road bridges may be overloaded not only by traditional heavily-loaded trucks and special vehicles, but also by abnormal extraordinary traffic loads of heavy industrial, construction, powerhouse and way equipments.

The analysis of climatological, aerodynamic and mechanical data of the existing transmission lines has shown that approximately 70% of their failures are caused by severe wind and ice storms (Gorokhov et al. 1997). These natural extraordinary events provoke sudden failures of not only deteriorated but also undamaged poles and portals.

In spite of the short duration of extreme events, they belong to persistent design situations characterized by random non-stationary safety margin processes. Therefore, any engineering possibility to ensure, assess and predict objectively the reliability indices of eccentrically loaded members is hardly translated into construction reality using the traditional semi-probabilistic methods of partial factors designed (in Europe) or load and resistance factors design (in the USA). The reliability level of compression members designed by these semi-probabilistic methods may be differ considerably (Diniz 2005; Jankovski and Atkočiūnas). Besides, in some cases, semi-probabilistic approaches may lead engineers to groundless overestimations or understimations of structures exposed to extreme variable traffic and climate actions.

Regardless of efforts to improve and modify semi-probabilistic approaches, it is inconceivable to fix correct values of reliability indices of members and their systems. Therefore, it is more expedient to base the structural safety design of members of annular cross sections on probability-based approaches (Krishnasamy 1987; Ranganathan and Borude 2001). These approaches allow us quantitatively assess uncertainties of resistances and action effects. Besides, the time-dependent reliability index design may enable structural engineers to make easier and more accurate selections of optimal unambiguous solutions.
The estimation of time-dependent extreme actions and the prediction of long-term safety of load-carrying structures are connected with some methodological and mathematical difficulties. Since the fairly exact and simple probabilistic assessment or prediction of survival probabilities of structures is hardly available in design engineering practice, many approximate approaches are considered and developed all over the world.

The time-dependent failure or survival probability of structures can be assessed and predicted by several diverse probability-based approaches and simplifications. Improved computational methods are based on the importance and conditional sampling procedures (Mori and Ellingwood 1993; Merkevičiūtė and Atkočiūnas 2006), direction simulation approaches (Ditlevsen 1997), variable-complexity approaches (Burton and Hajela 2003) and equivalent extreme-value events (Li et al. 2007). These approaches help researchers reduce sophisticated computational procedures and develop the analysis of structural optimization issues. However, the above mentioned improvements of mathematical models are inconvenient for structural designers and therefore not effective. These directions are hardly translated into everyday design reality.

The object of this paper is to call the attention of engineers and researchers to an application of reinforced concrete structures of annular cross sections and to propose for their analysis the unsophisticated reliability index design based on the method of transformed conditional probabilities.

2. Responses of particular members

For buildings, towers and bridges, the structural members (columns, poles, pier shafts) of annular cross section reinforced by steel bars uniformly distributed throughout their perimeters are represented in design practice by their particular members (normal or oblique sections and connections). Robustness and structural safety requirements of design codes and standards should be satisfied for all particular members of structures. Multicriteria failure models and survival probabilities of structural members may be objectively assessed and predicted only knowing mechanical and statistical parameters of their particular members.

The responses of compression particular members of annular cross sections with bending moments may be based on a plane cross section hypothesis and bi-linear concrete stress-strain relation (Fig 2b) or on a plastic method of the analysis of bending members with concentrical forces (Fig 2c) (Kudzys and Kliukas 2008, 2009; Kudzys et al. 2010).
When the eccentricity ratio \( e/r_s \leq 1 \), according to the design model presented in Fig 2b, the internal resisting compressive force and the resisting bending moment of this force about the centre point of annular cross section are:

\[
N_R = \left( k_c f_{cc} A_c + k_s \sigma_s A_s \right) \frac{r_s}{e + r_s} \quad (1)
\]

\[
M_R = N_R e \quad (2)
\]

where the response factors of resistances of concrete and reinforcement components may be presented in the forms:

\[
k_c = 1 - \frac{0.3 e/r_s}{1 + 10 \rho} \quad (3)
\]

\[
M_R = N_R e \quad (4)
\]

The compressive strength of concrete in reinforced members is:

\[
f_{cc} = a_{cc} a_2 f_c \quad (5)
\]

where its reduction factors due to effects of permanent compressive force \( N_p \) and concreting process are:

\[
a_{cc} = 1 - \Omega N_p / N_{\Sigma} \quad (6)
\]

\[
a_2 = 0.85 \cdot 1.7 \rho = 0.85 \cdot 1.7 A_s / A_c \quad (7)
\]

The statistics of resistance \( R_N \) of members exposed to compression with bending moments may be presented in the forms:

\[
R_{Nm} \approx \left( k_{ccm} f_{ccm} A_{cm} + k_{scm} \sigma_{scm} A_s \right) \frac{r_s}{e + r_s} \quad (8)
\]

The values presented in braces of Eq. (9) may be omitted if the approximation coefficient of variation \( \delta f_{sc} = 0.16 \) is applied.

The ultimate compressive stresses (MPa) in reinforcing bars of eccentrically loaded members may be expressed as:

\[
\sigma_{sc} = \varepsilon_{sc} E_s = 4.52(1.36 + 4p) \cdot 10^2 \quad (10)
\]

where \( p = A_s / A_c \) is the reinforcement ratio.

According to the design model demonstrated in Fig 2, c, the resisting bending moment, \( M_R \), of particular members reinforced by prestressed steel bars may be calculated by the universal equation defined by (Vadaliga 1979). It is presented in the form:

\[
M_R = 1.2 r_s \left( f_s A_s + N_R \right) \times
\left[
1 - \frac{f_s A_s + N_R}{f_{sc} A_s + (f_{sc} + f_{sc} - \sigma_p) A_s}
\right] \quad (11)
\]

where \( f_{sc} \) and \( f_{sc} \) are the strength in tension and compression of steel bars; \( \sigma_p \) is their prestress. The mean values of the strengths of mild and nonpressed high-strength reinforcements are: \( f_{sm} = f_{scm} = f_{scm} \) and \( f_{sm} = 500 \) MPa, \( f_{scm} = 600 \) MPa. Their coefficients of variation are:

\[
\delta f_{sc} - \delta f_{sc} = \delta f_{sc}^2 + \delta f_{sc}^2 / 2 = 0.10 - 0.15, \quad \text{where the components } \delta f_{sc} = 0.06 \text{ or 0.09 and } \delta f_{sc} = 0.08 \text{ or 0.12}
\]

define the statistical deviations and errors of right-angled epures of stresses.

For design practice, Eq. (11) may be rewritten in the form:

\[
M_R = T_1 T_2 / T_3 \quad (12)
\]

where

\[
T_1 = f_{sc} A_s + (f_{sc} + f_{sc} - \sigma_p) A_s \quad (13)
\]

\[
T_2 = 1.2 r_s \left( f_{sc} A_s + N_R \right) \quad (14)
\]

\[
T_3 = f_{sc} A_s + (f_{sc} - \sigma_p) A_s - N_R \quad (15)
\]

For nonprestressed members, the stress \( \sigma_p = 0 \). The statistics of resistance \( R_M = R_N \) are of the forms:

\[
\frac{\sigma^2 R_M}{R_{Nm}} \approx \left( \frac{2 T_2 T_3}{T_{1m}} \right) \times
\left[
A^2_{scm} \sigma^2 f_{ccm} + f_{scm}^2 \sigma^2 A_s + 
+ A^2_{scm} \sigma^2 f_{scm} + \left[A_s T_{sm} (1.2 r_s T_{sm} - T_2) / T_{1m}^2 \right] \sigma^2 f_{sc} + 
+ \left[A_s T_{sm} (1.2 r_s T_{sm} - T_2) / T_{1m}^2 \right] \sigma^2 N_{\Sigma} \right] \quad (16)
\]

with the variances of variables as follows:

\[
\sigma^2 f_{cc} = (\delta f_{cc} / \sigma_{ccm})^2, \quad \sigma^2 A_s = (\delta A_s / \sigma_{scm})^2, \quad \sigma^2 f_{sc} = (\delta f_{sc} / \sigma_{scm})^2, \quad \sigma^2 f_{sc} = (\delta f_{sc} / \sigma_{scm})^2, \quad \sigma^2 N_{\Sigma} = 2 \sigma^2 N_i.
\]
3. Second order effects

The first-order eccentricity of total compressive force $N_{\Sigma}$ of frame columns and poles or pier shafts respectively are:

$$e_o = M_{\Omega \Sigma}/N_{\Sigma} + e_{sh}$$  \hspace{1cm} (18)

$$e_o = l/\alpha + e_{sh}, \; \alpha \geq r_1/15 \text{ and } \geq 20 \text{ mm} \; (19)$$

where $M_{\Omega \Sigma}$ is the first order bending moment caused by total compressive force $N_{\Sigma}$; $l$ is the length of members; $\alpha = 500$ or 400 for their execution imperfections; $e_{sh}$ is associated with shifts of bearings (EN 1992-1 2004).

In design practice, the second order eccentricity caused by destroying compressive, longitudinal or lateral forces and structural deformations may be presented in the form:

$$e = e_o \eta + e_L$$  \hspace{1cm} (20)

where $e_o$ is defined by Eqs. (18) or (19),

$$\eta = \frac{N_B}{N_{\Sigma}} + \frac{\pi^2 \beta^2}{\beta - 1} \frac{N_{\Sigma}}{N_B - N_{\Sigma}}$$  \hspace{1cm} (21)

is the second order factor for definition of the bending moments resulting form a linear analysis of structures;

$$e_L = Q_L l^3 / (3EI)$$  \hspace{1cm} (22)

is the displacement of a member caused by longitudinal or lateral variable force $Q_L$ (Fig 3).

The buckling load of members, $N_B$, is expressed in terms of its effective length $l_e$ and flexural stiffness

$$EI = K_e E l_e I_e + E_s I_s$$  \hspace{1cm} (23)

where $I_e = \pi \left(r_2^4 - r_1^4\right)/4$,

$$\delta l_e \approx \delta A_e = (1.2 - r_1)[150(r_2 - r_1)];$$

$$K_e \approx 0.25(1 + \varnothing M_{OP}/M_{\Omega \Sigma})$$  \hspace{1cm} (24)

is the factor of concrete cracking and creep effects, where $\varnothing=1.2-2.0$ is the basic value of concrete creep ratio; $M_{OP}$ and $M_{\Omega \Sigma}$ are the first-order moments caused by permanent and total loads. Thus, the mean value and variance of buckling loads follow from:

$$N_{Bm} = \pi^2 (EI)_{m}/l_{om}^2$$  \hspace{1cm} (25)

$$\sigma^2 N_B = \left(\pi^2 E c_m l_{cm}^2/l_{om}^2\right)^2 \sigma^2 K_e +$$

$$+ \left(\pi^2 K_{cm} E c_m l_{cm}^2/l_{om}^2\right)^2 \sigma^2 E_c + \left(\pi^2 K_{cm} E c_m l_{cm}^2/l_{om}^2\right)^2 \sigma^2 l_e$$

where the statistical parameters are:

$$(EI)_{m} = K_{cm} E c_m l_{cm} + E_s I_s$$  \hspace{1cm} (27)

$$K_{cm} = 0.25(1 + \varnothing M_{OP}/M_{\Omega \Sigma})$$  \hspace{1cm} (28)

$$\sigma^2 K_e = \left[0.25(M_{OP}/M_{\Omega \Sigma})\right]^2 \left(\sigma^2 M_{OP} + \sigma^2 M_{\Omega \Sigma}\right)$$  \hspace{1cm} (29)

$$\sigma^2 E_c = (6E_e_x E_{cm})^2 \approx (0.15E_{cm})^2;$$

$$\sigma^2 l_e = (6I_e \times I_{cm})^2; \; \sigma^2 I_s = (6I_o \times l_{om})^2 \approx (0.1l_{om})^2.$$  

The coefficient $\beta = \pi^2 / e_o$ used in Eq. (21) depends on distributions of bending moments ($e_o = 8, 9.6$ and 12 for constant, parabolic and triangular distributions). For members without transverse loads, the coefficient $\beta = 1$ is normally a reasonable simplification. Therefore, the mean value and variance of second order factor $\eta$ by Eq. (21) may be expressed as follows:

$$\eta_m = l(1 - N_{\Sigma m}/N_{Bm})$$  \hspace{1cm} (30)

$$\sigma^2 \eta = \frac{1}{[N_{Bm} - (\theta_N N_{\Sigma m})]} \left[(\theta_N N_{\Sigma m})^2 \sigma^2 N_B +$$

$$+ N_{Bm} \sigma^2 \right]$$  \hspace{1cm} (31)

where the statistics $N_{Bm}$ by Eq. (25) and $\sigma^2 N_B$ by (26); $\theta_N$ represents the uncertainty of total compressive force $N_{\Sigma}$.

The mean and variance of second order eccentricities by Eq. (20) may be expressed as:

$$e_m = \epsilon_{om} \eta_m - e_{Lm}$$  \hspace{1cm} (32)

$$\sigma^2 e = \epsilon_{om}^2 \sigma^2 \eta + \eta_m^2 \sigma^2 \epsilon_o + \sigma^2 e_L$$  \hspace{1cm} (33)

Thus, the recommended equations give a possibility to calculate the statistics of second order eccentricities of compressive forces in the simple manner.

4. Time-dependent safety margins

According to full-probabilistic approaches, the performance process of particular members as their random time-dependent safety margin function may be presented in the form:

$$Z(t) = g[X(t), \theta(t)]$$  \hspace{1cm} (34)

where $X(t)$ is the vector of time-dependent processes of the basic (physical) variables representing the uncertainties in the material, action, geometrical and mechanical
models, and $\mathbf{\theta}(t)$ is the vector of processes of additional variables characterizing the models which give the values of resistance, $R$, and action effects, $E$, and may include the uncertainties of their probability distributions (JCSS 2000).

In the context of the analysis of particular members exposed to recurrent and coincident action effects, the time-dependent non-stationary process (34) may be recast more conveniently as follows:

$$Z(t) = \theta_R R(t) - \theta_G E_G(t) - \theta_Q E_Q(t) - \theta_S E_S(t) - \theta_W E_W(t) - \theta_L E_L(t)$$

where $R$ is the response by (1) or (11); $E_G$, $E_Q$, $E_S$, $E_W$, and $E_L$ are the action effects caused by permanent ($G$), floor sustained ($Q_f$) and extraordinary ($Q_e$), snow ($S$), lateral wind ($W$) and longitudinal (breaking or broken-conductors) ($Q_L$) actions (Fig 4a).

According to the test data (Vadlūga 1979), the means and standard deviations of additional variables $\theta_R$ of member resistances as forces or moments are:

$$\theta_{RM_n} = 0.987 \approx 0.99, \quad \sigma_{RM_n} \approx 0.08$$

and

$$\theta_{RM_m} = 1.016 \approx 1.02, \quad \sigma_{RM_m} \approx 0.08$$

for compression and flexural members, respectively. The statistical parameters of action effects may be defined as: $\theta_{im} = 1.0$ and $\sigma_{im} = 0.10$ (Rosowsky and Ellingwood 1992; Hong and Lind 1996; Vrowenvelder 2002).

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**Fig 3.** Modeling of eccentricities for columns (a), pier shafts (b) and poles (c)

**Fig 4.** Actual (a) and applied (b) models for time-dependent safety analysis of particular members
The annual extreme sum of sustained and extraordinary action effects of columns \( E_Q = E_Q(t) + E_{Qs}(t) \) may be modeled as a rectangular pulse renewal stationary process described by Type 1 (Gumbel) distribution of extreme values with the coefficient of variation \( \delta E_Q = 0.58 \) and the mean \( E_{Qm} = E_{Qm} = 0.47 \delta E_{Qk} \), where \( E_{Qk} = E_{Q,k} + E_{Q,k} \) is its characteristic value.

It is proposed to model the stationary processes of annual extreme snow and wind action effects by Gumbel distribution (Ellingwood 1981; JCSS 2000). The mean values and coefficients of variation of action effects \( E_S \), \( E_W \) and \( E_L \) are:

\[
E_{Sm} = E_{Sm} / \left[1 + \beta_{0,99}\delta E_S\right], \delta E_S = 0.3-0.7;
\]
\[
E_{Wm} = E_{WM} / \left[1 + \beta_{0,99}\delta E_W\right], \delta E_W = 0.2-0.5 \text{ and }
\]
\[
E_{Im} = E_{Im} / \left[1 + \beta_{0,99}\delta E_I\right], \delta E_I = 0.16 \text{ (Kudzys 2006)},
\]
where \( \beta_{0,99} \) is the 0.98 – fractile (quantile) of a standardized Gumbel distribution.

The extreme loads of structures of buildings and civil engineering works belong to persistent design situations in spite of short period of extreme events. The durations of intermittent extreme actions is considered as deterministic values. These durations are: \( d_Q = 1-14 \) and 1-3 days for merchant and other buildings, \( d_S = 14-28 \) days and \( d_W = 8-12 \) hours, \( d_L = 1-2 \) days. The concurrence of extreme snow and wind action effects is physically inconceivable.

The recurrence number of two concurrent extreme live and climate action affects \( E_1 \) and \( E_2 \) (Fig 4b), during the working life of structures \( t_n \), may be calculated by the formula:

\[
n_{12} = t_n (d_1 + d_2) \eta_1 \lambda_2 \quad (36)
\]

where the renewal rates of extreme events are \( \lambda_j = 1/\text{year} \) for snow and wind actions on ice-free surfaces of structures and conductors; \( \lambda_i = (0.05-0.5)/\text{year} \) for wind action on ice-covered surfaces; \( \lambda_{LE} = (0.05-0.2)/\text{year} \) for extreme longitudinal forces (Kudzys 2006). Thus, the recurrence numbers of extreme action effects during the period \( t_n = 50 \) years are: \( n_{Q,S} \approx 2.5, n_{Q,W} = 0.2-2.0, n_{L,W} = 0.01-0.07.\)

According to Ellingwood (1981), ISO 2394 (1998) and Kudzys (2006), Gaussian distribution law is to be used for resistance of compression reinforced concrete members and their permanent action effects. Lognormal, gamma or Weibul and exponential distributions may be used for long-term live (sustained), \( E_S \), and short-term (extraordinary), \( E_S \), variable action effects (ISO 2394 1988; JCSS 2000). The probability distribution of gravity and longitudinal variable live action effects of road bridges obeys a lognormal distribution (Kudzys and Kliukas 2008). When the coefficients of variation of load parameters are small (≤ 10%) or non-fatigue loadings are considered, for simplicity of probabilistic verifications, Gaussian distribution law may be used for non-extreme variable actions and their effects (EN 1990:2002).

5. Instantaneous survival probability

The extreme action effects caused by live \( (E_Q) \), snow \( (E_S) \), wind \( (E_W) \) and longitudinal \( (E_L) \) actions or extraordinary traffic loads may be treated as stationary random variables. Therefore for the sake of simplified but quite exact probabilistic analysis of eccentrically loaded members, it is expedient to present Eq. (35) in the form as follows:

\[
Z_k = R_c - E_k, \quad k = 1, 2, \ldots, n \quad (37)
\]

where \( k \) is the considered cut of the stationary process \( Z_k \); \( n \) is the number of extreme events by Eq. (36):

\[
R_c = 0_R K - 0_G E_G \quad (38)
\]

is the conventional resistance of a particular member the bivariate probability distribution of which may be modelled by a Gaussian distribution; \( E_k \) is the conventional bivariate action effect the combination of which depends on types of structures and their loading situations. It may be treated as interrupted rectangular process. Thus, for building columns, bridge pier shafts and transmission line poles, this action effect may be defined, respectively, as:

\[
E_{bc} = 0_Q E_Q + 0_W E_W \text{ or } E_{bc} = 0_Q E_Q + 0_S E_S \quad (39)
\]

\[
E_{psh} = 0_Q E_Q + 0_L E_L \quad (40)
\]

\[
E_p = 0_W E_W + 0_S E_S \quad (41)
\]

The mean and variance of conventional action effects may be calculated by following equations:

\[
E_m = 0_{in} E_m + 0_{jm} E_{jm} \quad (42)
\]

\[
\sigma^2 E = 0_{in}^2 \sigma^2 E_i + 0_{jm}^2 \sigma^2 E_j + 0_{in}^2 \sigma^2 E_i + 0_{jm}^2 \sigma^2 E_j \quad (43)
\]

\[
R_{cm} = R_{cm} - R_{cm} E_{Gm} \quad (44)
\]

\[
\sigma^2 R = 0_{cm}^2 \sigma^2 R + R_{cm}^2 \sigma_0 R + 0_{cm}^2 \sigma^2 E_G + 0_{cm}^2 \sigma^2 E_G \quad (45)
\]

where the mean \( R_m = R_{cm} \) by Eq. (8) and variance \( \sigma^2 R = \sigma^2 R_N \) by Eq. (9) for members exposed to compression with a bending moment; \( R_m = R_{cm} \) by Eq. (16) and \( \sigma^2 R = \sigma^2 R_M \) by Eq. (17) for members exposed to bending with a centrical force.
The instantaneous survival probability of particular members of considered structures at time \( t \), assuming that they were safe at time less than \( t \), may be expressed as:

\[
P[S(t)] = P[Z(t) > 0] = \int_{1/|X(t)|}^{\infty} f_{X(t)}(x) \, dx \tag{46}
\]

The stationary conventional resistance, \( R_c \), by Eq. (38) and the extreme action effects, \( E \), by Eqs. (39)–(41) may be treated as stochastically independent variables. Thus, the instantaneous survival probability of members may be represented by the single integral form:

\[
P(S_k) = P(R_c - E_k) > 0 \Leftrightarrow \int_{1/|R_c(-E_k)|}^{\infty} f_{R_c(-E_k)}(x) \, dx \tag{47}
\]

where \( f_{R_c(-E_k)}(x) \) is the density function of their resistance,

\[
F_E(x) = \exp \left( - \exp \left( \frac{E_m - x}{0.7794 \sigma E} - 0.5772 \right) \right) \tag{48}
\]

is the cumulative distribution function for their single or coincident extreme action effects and

\[
F_E(x) = \Phi \left[ \ln x - \ln (E_m) + 0.5 \ln \left( 1 + \sigma^2 E^2 / E_m^2 \right) \right] / \sqrt{2} \tag{49}
\]

for their joint variable traffic action effect

\[
E = \theta_Q E_Q + \theta_L E_L \tag{50}
\]

the probability distribution of which obeys lognormal law.

When extreme loading events are provoked by two stochastically independent action effects \( E_1 \) and \( E_2 \) (Fig 4b), a failure of structures may occur not only in the case of their coincidence but also when the value of one out of two effects is extreme. Therefore, three stochastically dependent safety margins of particular members should be considered as follows:

\[
Z_{1k} = R_c - E_{1k}, \quad k = 1, 2, ..., n_1 \tag{51}
\]

\[
Z_{2k} = R_c - E_{2k}, \quad k = 1, 2, ..., n_2 \tag{52}
\]

\[
Z_{12} = R_c - E_{1k} - E_{2k}, \quad k = 1, 2, ..., n_{12} \tag{53}
\]

where \( n_{12} \) is defined by Eq. (36). Therefore, from Eq. (47), three values of instantaneous survival probabilities \( P(S_{1k}) \), \( P(S_{2k}) \) and \( P(S_{12k}) \) are considered.

6. Long-term survival probability

When the safety margin of a member depends on combinations of uninterrupted and independent action effects, its long-term survival probability in one survival mode is equal to:

\[
P(S_n) = P(T \geq t_n) = P(Z_k > 0) = P(S_k) \tag{54}
\]

Thus, the long-term and instantaneous survival probabilities of such particular members are identical.

The stochastically dependent instantaneous survival probabilities of particular members exposed to interrupted extreme action effects form series systems. It is impossible to avoid the complicated intersections of recurrent failure events characterized by system elements. However, these probabilities may be calculated by unsophisticated method of transformed conditional probabilities (Kudzys and Lukoševičienė 2009, 2010). Since considered systems consist of equireliable and equicorrelated elements, the long-term survival probability of columns, shafts and poles may be expressed as:

\[
P(S_n) = P(T \geq t_n) = P(S_k) \left[ 1 + \rho_{kl} \left( \frac{1}{P(S_k)} - 1 \right) \right]^{n_m} \tag{55}
\]

where \( P(S_k) \) is defined by Eq. (47); \( \rho_{kl} \) is the correlation factor of a system the bounded index of which is:

\[
x = P(S_k) \left( \frac{4.5}{1 - 0.98 \rho_{kl}} \right)^{0.5} \left[ \frac{1 - P^2(S_k)}{1 - P^2(S_k)} \right]^{0.5 \rho_{kl}} \tag{56}
\]

The coefficient of correlation of system element is:

\[
\rho_{kl} = \frac{\text{Cov}(Z_k, Z_l)}{\sigma Z_k \times \sigma Z_l} = \frac{1}{1 + \sigma^2 E / \sigma^2 R_c} \tag{57}
\]

where \( \sigma^2 E \) is defined by Eqs. (39)-(41); \( \sigma^2 R_c \) – by Eq. (45).

When three safety margins of particular members from Eqs. (51-53) are taken into account, the total survival probability of members may be presented in the form:

\[
P(S_n) = P_1(S_n) \times P_2(S_n) \times P_3(S_n) \left[ 1 + \rho_{21} \left( \frac{1}{P(S_n)} - 1 \right) \right] \times \left[ 1 + \rho_{3,2} \left( \frac{1}{P_2(S_n)} \right) \right] \tag{58}
\]

where the ranked survival probabilities \( P_1(S_n) > P_2(S_n) > P_3(S_n) \) are defined by Eq. (55); \( \rho_{21} \) and \( \rho_{3,21} = 0.5(\rho_{31} + \rho_{32}) \) are the coefficient and factor of correlation.

A practical design of structures may be carried out using reliability index design approaches. The generalized reliability index of particular and structural members or their systems may be introduced as time-dependent value:

\[
\beta(t) = \Phi^{-1} \left[ P[S(t)] \right] \tag{59}
\]
where $\Phi(\bullet)$ is the cumulative distribution function of the standard normal distribution.

The target standard reliability index, $\beta_T$, of structures depends on their reliability classes but not on their destination. According to (EN 1990 2002; Ellingwood and Tekie 1999), for persistent design situations during a 50 year reference period, the target value for building columns should be equal to 3.8 and 3.2, respectively.

For bridge pier shafts, the index $\beta_T$ may be selected equal to 4.0 (Szerszen et al. 2005). It is recommended to use the indices $\beta_T$ equal to 2.9, 3.1 and 3.3 for tangent, angle of long-span and anchor or dead-end poles of overhead transmission high-voltage lines, respectively (Kudzys 2006).

The standard differentiation in the reliability of structures is based only on classes of failure consequences. However, the methodology of sustainable durability predictions requires to take into account future repair and replacement abilities of structural members. Therefore, it is expedient to make more precise the above-mentioned target values of reliability indexes and to relate with these abilities.

### 7. Revised survival probability

The members and their systems of existing buildings, bridges and power lines may be overloaded due to unfavourable over-increasing or abnormal service, traffic and climate actions. These overloads of structures may provoke sudden and unexpected failures. There are some limited attempts to transfer the approaches of semi-probabilistic limit state design to the structural quality revision of overloaded structures.

As it is known, extraordinary abnormal service and proof actions may lead to some reductions of uncertainties of resistances $R_N = N_R$ by Eq. (1) and $R_M = M_R$ by Eq. (11) and to correct instantaneous and long-term survival probabilities of members of existing structures. This correction can be carried put by methods based on the concepts of a truncated distribution and a Bayesian approach (Kudzys 2009). However, unfavourable action effects of particular members may be treated as an effective measure in the revised reliability prediction on existing structures when they are confirmed by quality statistical information data (Ellingwood 1996). Besides, the analysis of revised survival probabilities of structures by the truncated distribution and transformed Bayesian approaches leads approximately to the same results (Bulota and Kudzys 2010).

According to Madsen (1987), the revised instantaneous failure probability of particular members can be expressed as:

$$P_r(F_k) = P(Z_k < 0 \cap H > 0) / P(H > 0)$$  \hspace{1cm} (60)

where $Z_k$ is the random safety margin by Eq. (34); $H > 0$ is the event of inspection results showing a successful opposition of a member to unforeseen extreme action effect $E_{unf}$.

Two safety margins of particular members should be considered as: $Z$ by Eq. (34) and

$$H = \omega R - \theta G - \theta Q_S - E_{unf} > 0$$  \hspace{1cm} (61)

where $\omega = R_k / R_m = 1 - \beta_{0.99}$ is the ratio of characteristic and mean values of member resistances; $E_{unf} \geq 1.2E_{Q_k}$. The statistics of additional safety margin are:

$$H_m = \omega R_m - \theta G_m - \theta Q_{Sm} - E_{unf} > 0$$  \hspace{1cm} (62)

$$\sigma^2 H = \omega^2 \sigma^2 G + \sigma^2 G + \sigma^2 (Q_S, E_{Q_S})$$  \hspace{1cm} (63)

The survival probability of short-termly overloaded members may be expressed as:

$$P(H > 0) = \Phi(H_m / \sigma H)$$  \hspace{1cm} (64)

The revised failure probability of particular members by Eq. (60) may be transformed and rewritten in the form:

$$P_r(F_k) = \frac{P(Z_k < 0) \times P(H > 0) \left(1 - \rho_{2H} \right)}{P(H > 0)} = P(F_k) \left[1 - \rho_{2H} \right] = \left[1 - P(S_k) \right] \left[1 - \rho_{2H} \right]$$  \hspace{1cm} (65)

where the primary value of instantaneous survival probability $P(S_k)$ of members is calculated by Eq. (47).

### Conclusions

Eccentrically loaded structural members of annular cross sections are important load-carrying components of buildings, bridge piers and power transmission lines. Therefore, the structural safety of columns, pier shafts and poles should be assessed and predicted using up-to-date concepts of objective and explicit probability-based approaches including the method of reliability index design.

This method, better as the limit state design, help us use a strategy of time-dependent survival probability predictions and avoid both unexpected failures and unfounded premature repairs and strengthenings of structures subjected to recurrent extreme loading situations. However, analogically to traditional standard approaches of design codes, the reliability index design needs in simplifications of safety margin processes of particular members.

The time-dependent survival probability of particular members and their systems may be computed by the unsophisticated method of transformed conditional probabilities. Its approaches open quite a realistic way to implant the concept of reliability index design in the struc-
tural safety prediction of structures consisted of tubular or different type members.

It is shown that two resistances of tubular members of annular cross sections as compression and flexural structures may be considered in the reliability index design. In both cases, the second order factors may be expressed in terms of their flexural stiffness.

The revised values of survival or failure probabilities of members of overloaded existing structures may be defined by the transformed Bayesian approach.

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