PRACTICAL METHOD OF DESIGN FOR REINFORCED CONCRETE ELIPTIC SHELLS

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Abstract. The article deals with the problem of analysis of elliptic reinforced concrete shells when only membrane forces are taken into account. Normal and shear forces in shells in most cases are determined using stress function. Estimation of parameters for the stress function satisfying the edge conditions of the system is discussed in this paper.

Keywords: elliptic reinforced concrete shell, membrane stress and strain state, stress function, edge conditions.

Introduction

Thin-walled spatial structures (shells) are used in various areas of modern engineering. The general theory for analysis of shells was created and developed by scientists of various countries. Theoretical and experimental investigations in stress and strain state of shells cover various combinations of loads and actions, elastic and plastic deformations and various structural solutions as well. The majority of investigations are performed assuming that shells deform elastically. But there are executed research works where stress state in shells is considered as for an elastic plastic system (Karkauskas et al. 2004). In addition to static resistance of shells natural oscillation of shells is investigated (Grigorenko, Yaremchenko 2009).

In civil engineering thin-walled spatial structures may be load bearing structures of various civil engineering works or roofs for buildings and construction works. Elliptic shells in most cases are used to cover buildings with large internal spaces. This report deals with the problem of practical analysis for elliptical (of double positive curvature) shells.

Model for analysis of shell

Diagram of a shallow elliptic (of double Gaussian curvature) shell is shown in Fig 1.

Almost all points of such shell subjected to uniformly distributed load displaces in vertical direction equally. The shell curves only in the zones near the support contour, where the shell does not curve the bending moments and the shear forces do not appear. The major part of the shell is subjected to the membrane (without moments) stress and strain state. Suppose that the shell deforms elastically and it is possible to apply superposition principle, i.e. initially normal and tangential forces are determined according to the membrane theory for analysis of shells and then where it is required – bending moments and shear forces.

Fig 1. Diagram of shell

Determination of shell membrane forces

For determination of forces in elliptical shells only according to the membrane stress and strain state it is sufficient to find stress function. Assuming that shell support contour in its plane is absolutely stiff and out of its plane – flexible then normal forces at the support contour equal to zero. In such case stress function can be taken as a polynomial (Baikov et al. 1981, 1990).
\[ \varnothing(x, g) = \sum_{i=1}^{n} C_i \phi_i(x, y) = C_1 A_1 A_y + C_2 B_1 A_y + C_3 A_y B_1 + C_4 B_1 B_y, \]

\[ \text{Membrane (without moments) forces} \]

\[
\begin{align*}
N_x &= \frac{\partial^2 \Phi}{\partial y^2}; \\
N_y &= \frac{\partial^2 \Phi}{\partial x^2}; \\
S &= -\frac{\partial^2 \Phi}{\partial x \partial y}
\end{align*}
\]

Values of \( C_i \) are determined from a system of equations composed by the method of Bubnov–Galiorkin or of collocations. In the said engineering literature values of \( C_i \) for the shells of rectangular plan \((a \pm b)\) are determined only by the method of collocations. In this paper values of \( C_i \) obtained from a system of equations constituted by the Bubnov–Galiorkin method are expressed as follows:

\[
\begin{align*}
C_1 &= \frac{pR_1}{32a^3b^3} \varphi_1; \\
C_2 &= \frac{pR_1}{32a^3b^3} \varphi_2; \\
C_3 &= \frac{pR_1}{32a^3b^3} \varphi_3; \\
C_4 &= \frac{pR_1}{32a^3b^3} \varphi_4.
\end{align*}
\]

Values of coefficient \( \alpha_i \) are given in the Table 1 below.

<table>
<thead>
<tr>
<th>( \alpha_i )</th>
<th>Value</th>
<th>( \alpha_i )</th>
<th>Value</th>
<th>( \alpha_i )</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>0.64</td>
<td>( \alpha_7 )</td>
<td>106.52</td>
<td>( \alpha_13 )</td>
<td>2157.13</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>12.77</td>
<td>( \alpha_8 )</td>
<td>69.59</td>
<td>( \alpha_14 )</td>
<td>3148.57</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>45.28</td>
<td>( \alpha_9 )</td>
<td>18.95</td>
<td>( \alpha_15 )</td>
<td>16.60</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>66.31</td>
<td>( \alpha_{10} )</td>
<td>1.14</td>
<td>( \alpha_{16} )</td>
<td>57.38</td>
</tr>
<tr>
<td>( \alpha_5 )</td>
<td>19.61</td>
<td>( \alpha_{11} )</td>
<td>34.85</td>
<td>( \alpha_{17} )</td>
<td>83.57</td>
</tr>
<tr>
<td>( \alpha_6 )</td>
<td>74.34</td>
<td>( \alpha_{12} )</td>
<td>617.70</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For calculation convenience a table of \( \varphi_i \) values in relation to \( \lambda \) values is made.

In the particular case of a square plan shell when \((a = b)\) and \(R_1 = R_2 = R\) values

\[ \varphi_1 = 0.39329 \Rightarrow C_1 = 0.01229 \frac{pR}{a^6}; \]

\[ \varphi_2 = 1.24245 \Rightarrow \varphi_2 = 0.03883 \frac{pR}{a^{10}}; \]

\[ \varphi_4 = 18.7726 \Rightarrow \varphi_4 = 0.58665 \frac{pR}{a^{14}}, \]

and by the method of collocations when \( A(0; 0); B(0; 0.96); C(0.90; 0) \) or \( D(0.80; 0.86) \)

\[ C_1 = 0.00833 \frac{pR}{a^6}; \]

\[ C_2 = 0.0365 \frac{pR}{a^{10}}; \]

\[ C_4 = 0.353 \frac{pR}{a^{14}}. \]
Conclusions

Values of $C_i$ coefficients for stress function obtained by the method of Bubnov–Galiorkin suitable for analysis of elliptic square plan shells are presented in the paper. Application of the said coefficients simplifies evaluation of stress – strain state of such shells substantially.

References


Байков, В. Н.; Дроздов, П. Ф.; Трифонов, И. А.; Антонов, К. К.; Хлебной, Я. Ф.; Артемьев, В. П.; Рубинштейн, В. С. 1981. Железобетонные конструкции. Специальный курс [Baikov, V.N.; Drozdov, P.F.; Trifonov, I.A.; Antonov, K.K.; Hlebnov, J.F.; Artemjev, V.P.; Rubin- schtein, V.S. Concrete Structures Special course]. Moskva: Stroizdat. 767 p.