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EFFECTIVENESS, RELIABILITY AND SUBJECT RISK – SHAPING DRIVERS FOR THE SET OF POSSIBILITIES AND UTILITY FUNCTION WHEN INVESTMENT DECISION IS MADE UNDER UNCERTAINTY

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Abstract. The concepts of effectiveness, riskness and reliability are three cornerstones which together with utility of investor form the base for decisions perception and management logics in order to match the possibilities of investment space with investor's objectives. At this time risk, which is "*a chance or possibility of danger, loss, injury, or other adverse consequences*" (The Oxford Modern English Dictionary) or, specifically, in the area of investment management – "*the chance that an investment (as a stock or commodity) will lose value"* (Webster Dictionary) is the function of risksness of selected assets altogether with subject possibilities and skill to deal with the riskness. The paper will reveal a consistent way towards investment possibilities set description, when investment assets' possibilities are under uncertainty, what is understood in this paper as under stochasticity. As a possible mean of the above mentioned match the authors propose portfolio adequate for investment stochastic nature, presenting its formation and application principles. This model has broad application possibilities investing in exchange and capital markets, as well as forming sustainable investment strategies.

Keywords: utility, effectiveness, riskness, reliability, sustainability, risk.

1. Introduction

The concepts of effectiveness, riskness and reliability are three cornerstones which together with utility of investor form the base for decisions perception and management logics in order to match the possibilities of investment space with investor's objectives. The paper will reveal a consistent way towards investment possibilities set description, when investment assets' possibilities are under uncertainty, what is understood in this paper as under stochasticity. As a possible mean of the above mentioned match the authors propose portfolio adequate for investment stochastic nature, presenting its formation and application principles. This model has broad application possibilities investing in exchange and capital markets, as well as forming sustainable investment strategies.

The concepts of sustainable evolution or development, which have become the categories of analysis and management of countries, regions and other multiaspect and complex systems (Brauers, Ginevičius 2009; Čiegis *et al.* 2008; Čiegis *et al.* 2009a; Čiegis *et al.* 2009b; Diskienė *et al.* 2008; Misiūnas, Balsytė 2009), probably inherited their constructive philosophy and methodology about these systems' present and future necessity of harmonic interaction from the research of populations' (microorganisms, flora, fauna, etc.) sustainable development possibilities. However, such a conversion has certain inadmissible losses. One of the main attributes describing the sustainable development of populations – guarantee, that the current state or embraced trend of development will remain unchanged during a long period of time with high enough probability – is not cherished. The essence of this concept of persistence or survival is visualized by the provision that probability P of the changing state (for example, the probability that the quantity ξ of a certain population during a long enough period of time $t \in (0,T)$, will not drop lower than a certain value K_q, critical for the population) should remain at a certain level q:

$$P\left\{\xi_{t\in(0,T)} \geq K_q\right\} = q \qquad (1)$$

There is no doubt that analysis and management of the country and region sustainable development, related with research on the multidimensional processes, when separate aspects are linked with each other by the complex interdependencies, is a complicated problem, hardly conforming to operational management decisions. However, in many cases a provision about preservation of quantitatively measured guaranty can become a fundamental framework of the entire sustainable development nurturance.

The point already mentioned is very important in the projection of decisions for the large investment subjects, such as investment banks, mutual and pension funds, etc. An attempt of quantitative evaluation of the possible reliability or guarantee of activity results should mobilize the organizers of such strategies to reveal the problems, decisions of

which strongly influence company success, and which still do not have decisions giving satisfactory results (Rutkauskas *et al.* 2009).

The objective of the paper is to develop a sound investment decisions management conception based on main elements describing investment results: profitability, reliability and risk, and with the help of utility function.

The main task in order to achieve the stated objective is the selection of mutually compatible set of investment decisions and utility function.

A method of logical analysis of decision making under uncertainty, as well as individual experience of the authors has been used in order to perform the stated task and achieve the main objective.

2. The problems of efficiency, reliability and risk commensuration selecting investor's utility function

Selection of multicriteria functions and assessment of possibilities of practical application are highly important problems which receive much attention of mathematics, mechanics and other representatives of "quantitative" science. None the less attention should be given to these problems in social sciences, where great part of factors is generalized only qualitatively, and its direct application in multicriteria analysis provoke a lot of questions (Bivainis, Drejeris 2009; Ginevičius, Podvezko 2008a; Ginevičius, Podvezko 2008b; Ginevičius, Krivka 2009; Ginevičius, Zubrecovas 2009; Turskis *et al.* 2009).

This paper intensively analyses a problem of factors commensuration problem – detection of possibilities how much the change of one factor amount (unit) varies in terms of other factor certain amount (number of units), when it is attempted to retain the same weight of multicriteria function. Utility function and izoguarantees are examples of exceptional importance in this paper are.

2.1. Starting with commensuration of effectiveness and reliability

If already we agree that investment return possibilities ought to be given as set of possibilities {x} altogether with its probabilities, then the initial problem is how one can commensurate return with reliability or guarantee of the return, measured by survival function $F(x) = P\{\xi \geq x\}$. In this text we will speak having in mind that the set of possibilities is discrete. In any case the commensuration of return and its guarantee is made with the help of utility function.

Types, properties and application of utility functions in finance have been widely discussed in recent scientific literature (Andre 2009; Canakoglu, Ozekici 2009; Dostal 2009; Ehrgott *et al*. 2009; Kale 2009; Kliger, Levy 2009; Pennings, Garcia 2009; Yu *et al*. 2009). When we analyse already formed set of portfolios, then utility function is seen as some kind of multiplicative or additive (Feige 2009; Koksalan, Bilgin 2009), or other kind of function of return *e* (its mean value, the most expected value or any kind of quintile) and reliability *g* of chosen return *e* and riskness of set of possibilities or risk of subject - *r*:

$$U = u(e; g; r). \quad (2)$$

Commensuration of profitability and risk is strongly discussed in conceptual and practical arenas. But when we have only set of possibilities {x} and their guarantee {F(x)} – usually understood as survival function, then an amount of concepts about commensuration of return and guarantee is no less intensive. Let's use the most simple funcion under consideration for the role of utility measurement on these two parameters – *e* and *g*:

$$U = u(e, g) = \alpha_0 e^{\alpha_1} g^{\alpha_2}, \quad (3)$$

where α_i, i = 0, 1, 2 – are some parameters found in process of calibration of function.

Then usual picture for selection of optimal composition of possibility with its reliability one can see in Fig. 1. The concept of sub-utility 1 (further we will also use sub-utility 2 concept) is used for line that has equal utility for different combination of *e* and *g*.

This concept and technique of optimization and presentation will be used in three-dimensional case.

2.2. Evaluation of possibilities reliability – the core problem in investment decisions management

Evaluation of solutions reliability or guarantee is the urgent decision management (decisions selection and implementation) problem, that differentiates and at the same time associates logics and methods of solutions, gained under the terms of determinated relation, and solutions, gained under the terms of uncertainty and risk. Solutions reliability, under the terms of determinated relation, associates with the accuracy of relation measurement and the propriety of decision methods, while evaluation of reliability, under the terms of uncertainty and risk, assumes to be entirely distinctive problem. In the paper we will present portfolio, adequate to the evaluation of investment possibilities reliability, or simply adequate portfolio, which appears to be not only an innovative approach to

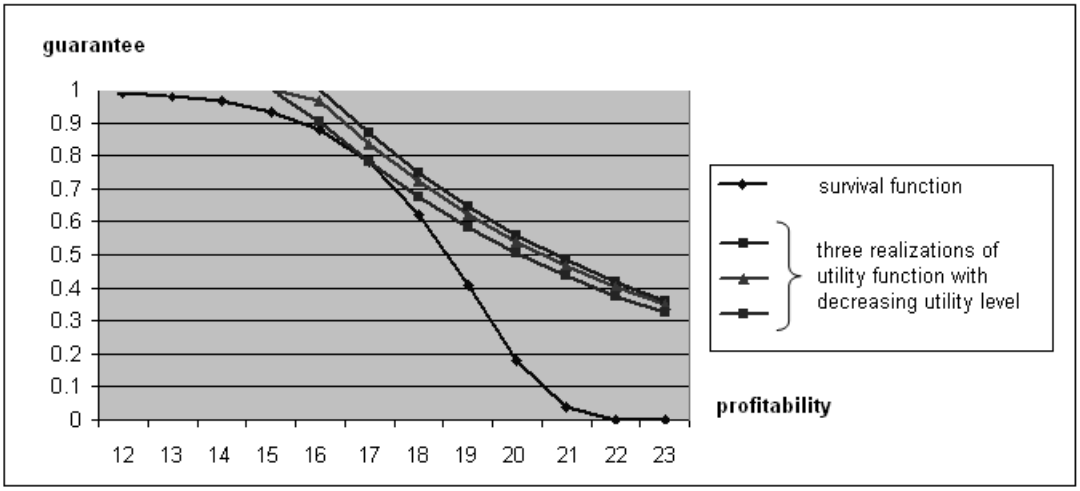


Fig. 1. Intersection of Sub-Utility 1 with Survival Function

investment decisions management, but also an effective mean to analyze possibilities and project sustainable development of sophisticated systems (Rutkauskas *et al.* 2009). Analyzing stochastic values or processes we will measure reliability of possibility as reliability or survival function $S(x) = 1 - F(x)$, here $F(x) = P\{\xi<x\}$ is accumulated distribution function of investment possibilities. Hence $S(x) = P\{\xi\geq x\}$.

2.3. The concept of iso-guarantee

In order to approach the evaluation of investment utility for investor according effectiveness, effectiveness possibilities' riskness and reliability of every effectiveness possibility, a concept of iso-guarantee is used, which was proposed by the authors (Rutkauskas 2003).

Iso-guarantee of investment portfolio is a line in "portfolio risk – portfolio profitability possibilities" plane, connecting possibility values of the same guarantee under changing risk conditions.

Here and further in the paper investment portfolio effectiveness will be considered as portfolio profitability.

In probability theory and mathematical statistics terminology, iso-guarantee should indicate a line, the projection of which in profitability-risk plane is a q-level quintile line

$$\xi_q^s : P\left\{\xi^s \geq \xi_q^s\right\} = q \qquad (4)$$

of the set of profitability possibility values ξ^s, when portfolio riskness (s – standard deviation) is changing (increasing).

In general case efficiency line of modern portfolio is not iso-guarantee. If portfolio profitability possibilities mean equals median for each risk level, then efficiency line becomes the iso-guarantee of 0,5 level.

According efficient frontier generation logics, if all the possible quintiles become profitability resultant, then structural formation of the set of iso-guaranties is presented in Fig. 2.

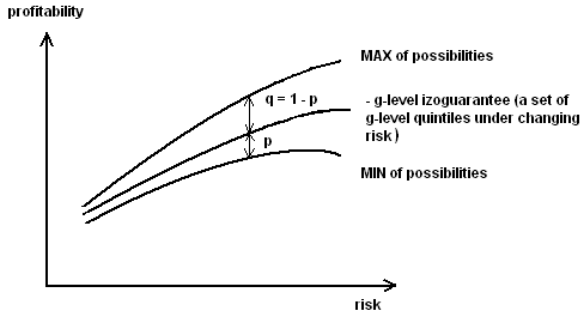


Fig. 2. The Anatomy of Iso-Guarantee

If all iso-guaranties from "risk-profitability" plane could be lifted into "risk-profitability-reliability (guarantee)" space according to their reliability level, the sphere of all possibilities of investment portfolio would be depicted as in Fig. 3.

On the iso-guarantee one can expertize with iso-utility 2 line for the commensuration of profitability and risk, understood as a function of profitability and risk. Usually the concept of indifference curves is used in such a case.

When iso-guaranties on the set of portfolio possibilities are determined, each investor should be interested in a portfolio which more useful for him. The selection is usually performed by sub-utility 2 function (Fig. 5). In modern portfolio case sub-utility is entitled as indifference curve or utility function keeping in mind that selection is in

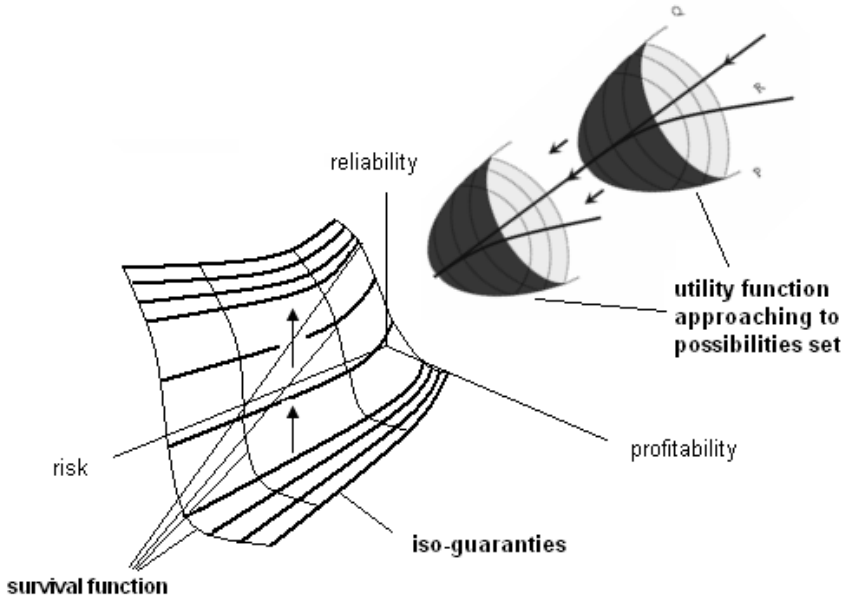


Fig. 3. Search for Possibility of Highest Utility, when Possibilities' Set and Utility Function are Stochastic Functionals of Profitability, Risk and Reliability

"risk-profitability" plane. Sub-utility 2 here is used because the term sub-utility 1 has already been used for selection of most useful situation in "profitability-guarantee (reliability)" plane.

2.4. Adequate portfolio as natural result of modern investment portfolio development, devoted to integral profitability, risk and reliability adjustment according investor's utility function

Function of fundamental modern (Markowitz) portfolio and its further amplifications (Fabozzi, Markowitz 2002; Reilly, Brown 2003) is a possibility to commensurate investment profitability and risk objectively and to give an opportunity of selecting a portfolio taking into consideration investor's indifference curve. Efficiency line of portfolio values is fundamental mean of such choice and optimization (Sharpe 1964). However, evaluation of the aimed profitability's reliability and along with general commensuration of profitability, risk, and reliability levels, compounding an effective zone in three-dimensional – profitability, risk, reliability – space is of premium importance for today's investor. Effective zone, that is compounded as an intersection of survival function of portfolio possibilities values and iso-guaranties, not only contributes for such a commensuration, but also becomes a set of constraints searching for the possibility of the largest profitability for an investor, in other words a criteria invoking his utility function, that depends on profitability, *risk,* and reliability. Here the word *risk* is distinguished in order to stress the principal difference between the riskness of investment possibilities and investor's risk, that depends also on individual features of an investor (Rutkauskas *et al.* 2009). In fact, risk, which is "*a chance or possibility of danger, loss, injury, or other adverse consequences*" (The Oxford Modern English Dictionary) or, specifically, in the area of investment management – "*the chance that an investment (as a stock or commodity) will lose value"* (Webster Dictionary) can be defined as the function of risksness of selected assets altogether with subject possibilities and skill to deal with the riskness. Thus, we try introducing our own concept of risk, which from slightly to drastically differs from variability of process or event that sometimes is used for quantitative measuring of risk.

3. Adequate portfolio formation considering investor's utility function

3.1. Formation of adequate portfolio

In order to reveal portfolio investment decisions reliability role mechanism in details, we will briefly take a look over adequate investment decisions reliability assessment portfolio anatomy.

Fig. 4 presents adequate portfolio for investment decisions reliability assessment. „Mean – standard

deviation" portfolio (modern, or Markowitz portfolio (Markowitz 1952)), shown in Fig. 4 section *a* is a portfolio formed for independent values, having normal probability distributions. Next, a bunch of the possible values of all possible "quintiles – risk" portfolios (Fig. 4, section *b*) is formed. More precisely speaking, not all the quintiles were used for this bunch here, but only deciles.

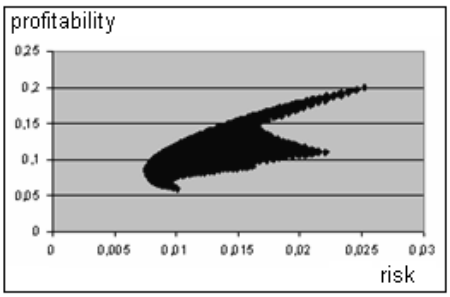
On the basis of the set of formed portfolio values and on the analogy of the reception of efficiency line of modern portfolio, a set of all efficiency lines is composed, which is called the efficiency zone. The efficiency zone is presented in Fig. 4, section *c*.

There is no doubt that investor is interested not only in quantitative indicators of investment profitability possibilities, but also in guarantee of each possibility i.e. probability that investment profitability (return) will not drop below the certain level. In case of modern stock portfolio, the guarantees of investment profit possibilities are usually not discussed, although in case when portfolio profitabilities' possibilities' distribution is a normal one, there is a direct possibility to evaluate these guarantees, if mean value and standard deviation are known (Rutkauskas 2000). Raising every efficiency line from the Fig. 4 section c by the level of its guarantee or reliability, the three-dimensional view (profitability, riskness, reliability) of the investment portfolio is formed (Fig. 4 section d).

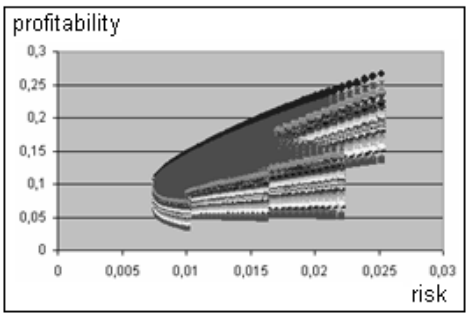
3.2. Throughout sub-utilities to utility function and investment decision-making

Let's have a set of all possible portfolios from selected range of assets. After survival functions are determined for each portfolio, the next step towards the efficient portfolio development is taken – one of the portfolios from the efficient set is selected. Every investor selects the portfolio that satisfies the investor's desired risk level and provides the maximum profitability under mentioned risk level (Hirt, Block 1993). Sub-utility 1 function helps the investor selecting optimal portfolio. Sub-utility function is viewed as the family of indifference curves, approaching the efficient frontier as the utility level decreases (Rutkauskas 2006). It is shown in Fig. 5.

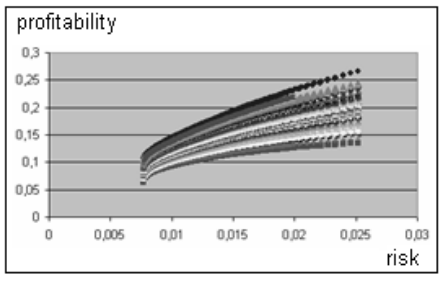
It is worth noticing that Fig. 5 presents only the family of indifference curves of a risk-averse investor. The reason for this is that portfolio theory was originally developed within the context of a risk-averse individual investor (Lumby 1994). After plotting the efficiency line and utility function, it is possible to find the most efficient portfolio for particular investor. The most efficient portfolio appears in that point of efficiency line, where indifference curve touches the efficiency.



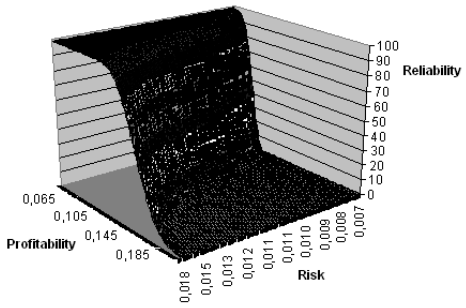
a) Markowitz or "Average-Standard Deviation" Portfolio



b) Bunch of "Quintiles–Standard Deviation" Portfolios



c) The Confidence Zone of Adequate Portfolio



d) Three-Dimensional View of the Investment Porfolio

Fig. 4. The Sequence of Adequate Portfolio Construction

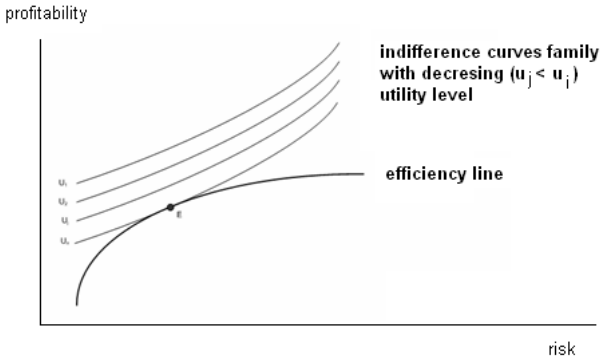


Fig. 5. The Intersection of Indifference Curve Family and Efficiency Line (E) – the Optimal Portfolio for the Investor in "Profit-Risk" Plane line.

After plotting the efficiency line and utility function, it is possible to find the most efficient portfolio for particular investor. The most efficient portfolio appears in that point of efficiency line, where indifference curve touches the efficiency line. In Fig. 5 it is point E. Such portfolio (i.e. two parameter (profitability average and riskness) portfolio's value indicator) maximizes investor's utility. Another investor, which has other acceptable risk level, would have other utility function, and, in turn, other optimal portfolio. If Fig. 1 presents the schematic view of the two-parametrical utility functions in "profitability-reliability" plane where they are titled as sub-utility 1, in „quintile-risk" plane (Fig. 5) they are titled as sub-utility 2. However, forming asset portfolio and plotting the utility function in two-dimensional plane is not fully informative, because investor does not get information about all three parameters of possibility: profit, reliability, risk (Rutkauskas *et al.* 2009). For this reason in Fig. 6 three-parametrical utility function in "profitability-risk-reliability" plane can be constructed (right side), approaching the adequate portfolio set of values, searching for the most useful portfolio values for the subject whose interests reflects the utility function.

Graphical view of such utility function is constructed according analytical function:

$$N = \alpha_1 \left(\frac{e}{r} \right)^{\alpha_2 g} \tag{5}$$

where:

N – utility level,

e – profitability,

r – risk,

g – guarantee,

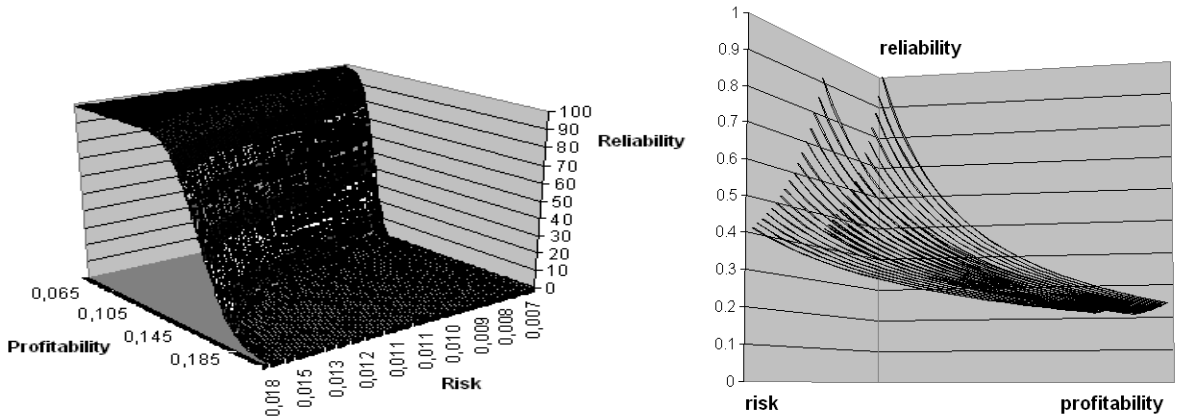
α_1, α_2 – some parameters of function 2, of course different for different investors.

Such specification of decision-making procedure is analytically meaningful because the authors solved the complex stochastical programming problem with the help of imitative technologies and graphical decision-making means.

Optimization of adequate portfolio is a solution of complex optimization problem. In case of complex probability distributions of investments profitability, their interrelations, and complex form of utility function, formation of optimal portfolio is complicated and has no universal methods for decision making. Thus, in order to have an operative mean of such decision, that is necessary for the application of adequate portfolio for decision making, one should use imitative technologies, which are almost the only mean of information supply for the quick decision management process.

4. Conclusions

Utility, riskness and reliability are three main attributes that ought to be used integrally when investment decision is under preparation.



a) Portfolio Set of Values b) Utility Function

Fig. 6. Three-Dimensional View of Portfolio Set of Values and One of Possible Utility Functions

Adequate investment portfolio seems to be theoretically sound and practically effective instrument for investment decision making in global capital and exchange markets.

Graphical visualization of decision search or utility maximization selected on the basis of stochastic optimization seems to be a reliable instrument in this case.

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