TOWARDS BRIDGE OPTIMIZATION: EVALUATION OF OPTIMAL PARAMETERS AT CRITICAL RESPONSE STATES

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Abstract. The paper is assigned to development of optimization techniques for load carrying structure of a bridge, constructed from standard steel profiles. Statement of a problem, involving constraints due to relationships of codified regulations in concert of an employment of elastoplastic response of structure ensures an obtaining an optimal structure compatible with actual engineering design. These techniques allows to employ a significant strength resource versus usual elastic design, id est result the significant material savings. Solution of such optimization problem is rather difficult even applying an iterative approach due to peculiarities of development of plastic deformations, yield conditions, involving several components of internal forces, stiffness constraints, evaluation of load combinations due to code regulations, finally, the problem size. Computational efficiency and successful iterative optimization (convergence) of structures, involving stiffness constraints (full optimization problem) highly depend on choosing starting point and that of an application of rational limitations for minimum values of designed parameters and that for extreme displacements. This separate problem by itself is difficult to solve also. The above mentioned values are identified by solving the limit equilibrium optimization and analysis problems. Then the full optimization problem can be solved. Such an optimization approach was successfully employed by authors for simpler structures.

Keywords: bridge, optimization, elastoplasticity, iterative solution, rational limits, starting point, convergence

1. Introduction

Transport engineering structures such as tunnels, bridges, flyovers, etc has significant influence to total cost of road network. Creation of economically efficient, safe structure actually is based on engineer experience. Usually design process realizes a certain structure project, obtained following the codified regulations. In fact, such an approach realizes the analysis problem solution, id est the determining of design values under prescribed all conditions: external actions, material, structure topology, soil response, etc. This investigation presents an approach finally leading to an obtaining of an optimal structure, following certain optimality criterion (e.g. minimum weight one) satisfying main codified requirements: strength, constructional requirements close to limit equilibrium state. The residual response to loading is realized with elastoplastic material model, compatible with steel structures. An evaluation of plastic properties leads to significant material savings versus usual elastic design (analysis) or even an optimization of the expected response in elastic way.

Solution of the full optimization (involving stiffness constraints) problem, being formulated directly, leads to essential numerical realization difficulties even for small-size structures due to an employment of the complementarity conditions [1-3]. Therefore the problems related with the real structures used to be solved iteratively. One must note that even an application of the iterative solution of the full optimization problem does not ensures the convergence of iterative solution process due to: peculiarities of development of plastic deformations, the evaluation of several components of internal forces when prescribing a stress state of structural members, evaluation of loading states due codified design, finally the size of problem, reaching thousands constraints for actual engineering structures.

The method for realizing of an iterative approach to optimize a structure via the three-staged stepped optimization techniques allows an avoiding the direct evaluation of above mentioned complementarity conditions and ensures the problem solution convergence and significant saving of computation resources. This method was developed by authors and was realized for truss-type structures subjected by load combinations [4], for orthogonal frames or trusses subjected by constant loading [5, 6]. The method currently is developed for combined structure (stress state is defined by several components of internal forces), id est pedestrian steel bridge, subjected by load combinations. Such an approach is actual in engineering practice, and has been analyzed by many authors [e.g. 7, 8, 9].

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It was proved by authors that computational efficiency and successful convergence of iterative optimization process highly depend on choosing of the optimal starting point (feasible elastoplastic state of structure) and on using of the rational limits (allowable minimum magnitudes) for design values of structural members and rational bounds of deforming of structure. The above mentioned values are obtained by solving an optimization problem in limit equilibrium state. After that the subsequent solution of analysis problem is performed to view a structural response of optimal structure being in the state just before the plastic failure.

The full optimization problem should be solved, applying the obtained results and involving the necessary displacement limitations (these generally being larger the admissible ones, following the codified requirements).

An approach to be realized corresponds to a holonomic structure, subjected by load combinations. An evaluation of non-holonomic structure, which deformable response depends on a loading history is more difficult and problematic even for small size structures [10, 11].

The optimization problem solution even in equilibrium state and the subsequent response analysis of obtained optimal structure are difficult by itself ones when the actual large-dimensional structure (e.g. considered in the paper) is under investigation.

In further investigations the full optimization problem will be solved by means of developed by authors iterative optimization approach [3], having expanded it for the considered steel bridge.

2. Mathematical model of full optimization problem

Discrete model of the load carrying structure of bridge is analyzed. Topology of structure, consisting of members with perfectly elastoplastic material standard profiles, is prescribed. Structure is subjected by the load combinations \(i = 1, \ldots, v, \ldots, \xi\), combining permanent \(q_k\) and live \(q_k\) loads due to codified regulations.

Let all members of structure to be designed from certain number of different standard steel profiles. Create the vectors of these members corresponding to cross-sectional areas \(A\) and corresponding limit plastic moments \(M_0\). Note that both values are functionally related.

The full optimization problem is formulated as follows: find a distribution of cross-sectional areas limit plastic moments \(M_0\), corresponding to minimum volume (or weight of structure) when its separate members (cross-sections) are plastically deformed. Displacement constraints are introduced to ensure structural deformable response within admitted bounds \(u_{\min} = (u_r + u_e) \leq u_{\max}^+\) \(i = 1, \ldots, v, \ldots, \xi\). Here \(u_{\min} > 0\) and \(u_{\max} < 0\) are the characteristic magnitudes of upper and lower bounds of displacements (displacement vector components), determined by requirements of design codes in concert with solutions of analysis problems. Following codified regulations the other artificial requirements (minimum slenderess, minimum cross-sectional area, etc) can be introduced in concert to above mentioned obligatory requirements. All requirements generally constrain the free development of plastic deformations, therefore the induced residual fields of inner forces \(S_r\) and displacements \(u_e\) are the ones ensuring all the above mentioned and other eventually introduced requirements.

Usually the introduced displacement constraints are dominating versus the strength (plastic collapse) and the other ones. One can reach the optimal structure at plastic collapse state (result compatible with the structural optimization in limit equilibrium state), when increasing the displacement limitation magnitudes sustainably. We remind the reader that the most optimal structure (at limit equilibrium state) versus any others, being in a state of plastic collapse, is obtained in this way.

The general mathematical model of full optimization problem reads:

\[
\text{Find } \quad L^T M_0 \rightarrow \min, \quad (1)
\]

\text{subject to:}

\[
[B] M_0 - [\Phi] S_r \geq [\Phi] S_e, \quad (2)
\]

\[
[A] S_r = 0. \quad (3)
\]

\[
[D] S_r + \sum_{i=1}^{\xi} [\Phi]^T \lambda_i - [A]^T u_e = 0. \quad (4)
\]

\[
\sum_{i=1}^{\xi} \lambda_i^T ([B] M_0 - [\Phi] (S_r + S_e)) = 0. \quad \lambda_i \geq 0. \quad (5)
\]

\[
u_{\min} \leq [P] (u + u_e) \leq u_{\max}, \quad (6)
\]

\[
M_0 \geq M_{0\min}, \quad i = 1, \ldots, v, \ldots, \xi. \quad (7)
\]

Here subscripts \(e\) and \(r\) denote elastic and plastic responses of structure, respectively. Vector of inner forces \(S = (\text{M}, \text{N})^T\) selects bending moments and axial forces, respectively. \(\lambda_i\) is a vector of plastic multipliers, corresponding the \(i\)-th load combination. The matrices \([A], [D], [\Phi]\) represent the coefficients of equilibrium equations, that of physical equations of elastic response and that of yield conditions. Matrices \([B]\) and \([P]\) are the configuration matrices of limit bending moments and displacements, relating the selected values with the ones of the whole discrete model of structure.
Equations (5) represent the complementarity conditions of the structure. Linear optimality criterion (1) can be easily transformed to minimum theoretical volume or weight of structure, as vector \( L \) represents total lengths of optimized members.

Solution of the mathematical model (1)-(7) directly or replacing it by staged iterative approach [6], excluding an employment of (5), necessitates: firstly, an identifying the optimal starting point, related with choosing primary cross-sectional profiles; secondly, an evaluation of actual displacements \( u_i = u_{ei} + u_r \) of the structure being of the state, close as possible to plastic collapse state of structure.

The limit equilibrium optimization and analysis problem are to be solved for this aim.

3. Limit equilibrium optimization problem formulation and solution peculiarities

The limit equilibrium optimization problem corresponds the state of structure at plastic collapse. Here the unknown values are the vector of limit bending moments \( M_0 \), the vectors of forces \( S_r \) and that of displacements \( u_r \), both corresponding to adapted residual state of structure versus all known elastic response values \( S_{ei} \) (known from elastic analysis of structure due to considered load combinations \( i = 1, \ldots, \zeta \)).

A solution of the problem requires to identify a functional relationship between values, prescribing elastic response (moment of inertia \( I \) and areas \( A \) of cross-sections, to be optimized) and limit bending moments \( M_0 = \sigma_A W_{pl} \). Here \( \sigma_A \) is material yield limit. The above mentioned relationships for standard European steel profiles, (to be employed for bridge design) can be expressed with sufficient accuracy via the following power functions:

\[
I = a_1 A^{b_1}, \quad \text{(8)}
\]

\[
W_{pl} = a_2 A^{b_2}. \quad \text{(9)}
\]

Here \( a_1, b_1, a_2 \) and \( b_2 \) are the simulated coefficients, depending on the certain profile class.

The linearized yield conditions of \( j \)-th cross-section (subsequently being expressed via yield matrix \([\Phi]\)) read:

\[
\begin{align*}
M_{0j} - M_j & \geq 0, \\
M_{0j} + M_j & \geq 0, \\
M_{0j} - M_j / 1.18 - c_j N_j & \geq 0, \\
M_{0j} + M_j / 1.18 - c_j N_j & \geq 0, \\
M_{0j} - M_j / 1.18 + c_j N_j & \geq 0, \\
M_{0j} + M_j / 1.18 + c_j N_j & \geq 0.
\end{align*}
\]  \( \text{(10)} \)

An eccentricity coefficient for tensile member taking into account (8) and (9) is defined by:

\[
c^+ = \frac{M_{0j}}{N_{0j}} = \frac{\sigma_A a_3 A^{b_3}}{\sigma_{\chi} A} = a_3 \cdot A^{b_3 - 1}. \quad \text{(11)}
\]

The eccentricity coefficient for compressive member must evaluate a buckling factor. The limit axial force under compression is \( N_{cr} = \chi \sigma_A A = \sigma_{cr} A \). Here the characteristic value of critical stress is \( \sigma_{cr} = \chi \sigma_A \), where \( \chi \) is reducing factor, depending on the dimensionless slenderness of the member. The slenderness is defined by:

\[
\chi = \frac{\lambda}{\lambda_E} \frac{l_b}{l_{Ec}} \frac{1}{i \cdot \lambda_E} = \frac{l_b}{\sqrt{l_{Ec}/A \pi E}} \frac{1}{\sqrt{\pi E/\sigma_A}}, \quad \text{(12)}
\]

where \( \lambda \) is member slenderness; \( l_b \) is member buckling length, evaluated following [12]; \( \lambda_E = \pi \sqrt{E/\sigma_A} \); \( E \) is elasticity modulus of material.

The reducing factor \( \chi \), following EN 3 [13], is defined by:

\[
\chi = \frac{1}{\varphi + \sqrt{\varphi} - \chi^2}, \quad \text{but } \chi \leq 1, \quad \text{(13)}
\]

\[
\varphi = 0.5 \left[ 1 + \alpha (\lambda - 0.2) + \lambda^2 \right]. \quad \text{(14)}
\]

Here \( \alpha \) is coefficient depending on profile class and lamination (hot or cold rolled) case.

Thus, by employing the above described relationships, the eccentricity coefficient for compressive member finally reads:

\[
c^- = \frac{M_{0j}}{N_{crj}} = \frac{\sigma_A a_3 A^{b_3}}{\chi \sigma_A A} = a_3 \cdot A^{b_3 - 1}. \quad \text{(15)}
\]

The codified design introduces a constraint of minimum slenderness \( \lambda_{lim} \) for structural members under compression. It, when being transformed in terms of optimized values of the mathematical model, corresponds the group (7) constraints. Thus, the minimum limit bending moment, corresponding the \( \lambda_{lim} \), is:

\[
\lambda_{lim} = \frac{\lambda_{lim}}{\lambda_E} \frac{1}{\chi} = \frac{\lambda_{lim}}{\lambda_E} \frac{1}{\sqrt{\pi E/\sigma_A}}. \quad \text{(16)}
\]
The limit equilibrium optimization problem, taking into account the above given relations, reads:

Find \[ L^T M_0 \rightarrow \min, \] subject to:

\begin{align*}
[B] M_0 - [\Phi] S_r & \geq [\Phi] S_{ri}, \quad (18) \\
[A] S_r &= 0, \quad (19) \\
M_0 &\geq M_{0,cr}, \quad i = 1, ..., \xi. \quad (20)
\end{align*}

As the elastic inner forces as well as the eccentricities depend on cross-sectional properties \((A, I)\), id est on the optimized parameters \(M_0\), the considered optimization problem is to be solved iteratively (via optimization loops), until the prescribed convergence criterion is satisfied. The main stages of optimization loop are:

1. Identifying primary minimum slenderness magnitudes \(\lambda_{\text{lim}}\) following codified regulations;
2. Identifying minimum limit bending moments \(M_{0,cr}^{\min}\) (see formula (16)) for members of selected profile classes;
3. Identifying primary magnitudes of eccentricities \(c^+\) and \(c^-\) for structural members (see formulae (11) and (15));
4. Creating matrices \([D], [\Phi]\);
5. Solving limit equilibrium optimization problem (17)-(20), obtaining limit bending moments \(M_0^{\text{iter}}\) for running optimization loop;
6. Applying the determined \(M_0^{\text{iter}}\) for identifying the components of the vectors \(A^{\text{iter}} = \left(M_0^{\text{iter}} / \sigma_{\gamma} \cdot a_1\right)^{b_1}\), corresponding the \(A^{\text{iter}}\) and then determining the moments of inertia \(I^{\text{iter}}\);
7. Checking convergence criterion: difference of limit bending moments of running \(M_0^{\text{iter}}\) and previous \(M_0^{\text{iter}-1}\) optimization loops is greater the prescribed admissible error \(\varepsilon\) (small enough magnitude):

\[ |M_0^{\text{iter}} - M_0^{\text{iter}-1}| > \varepsilon? \]

7.1 yes: \(A^{\text{iter}}\) and \(I^{\text{iter}}\) are fixed as input data for new optimization loop (stages 2 -7) to be started;
7.2 no: ending iterative optimization process, processing necessary output data.

The obtained optimal distribution of limit bending moments serves the lower bound \(M_0^{\text{min}}\) for full optimization problem (1)-(7) solution, id est for the one, containing displacement constraints.

4. Solution of analysis problem

The analysis problem for the above obtained problem is to be solved aiming to identify the upper and lower bounds of displacements \(u_{\text{adm}}\) and \(u_{\text{adm}}\). They subsequently will be employed for solving of the full optimization problem for structure, satisfying codified displacement constraints and responding to loading in elastoplastic way.

Note that the most optimal structure, obtained when including displacement limitations, is the one, possessing possibly more plastic deforming. If displacement constraints are too strict, plastic deforming can be not reached at all, if they are too soft, the solution leads to limit equilibrium optimization problem.

The analysis problem can be solved by introducing the considered load cases, corrected by minimal reduction factor \(\gamma_{\text{red}} > 1\), aiming to obtain a feasible solution of analysis problem. Following [5] the mathematical model of analysis problem for considered load combinations \(i = 1, ..., \nu, ..., \xi\) reads:

Find \[ \frac{1}{2} \sum_{i=1}^{\xi} \lambda_i \left[M_0 - [\Phi] [D]^{-1} \bar{\lambda}_i \times [\Phi] [I] [\Phi] \right] \rightarrow \min, \]

subject to \[ \lambda_i \geq 0. \]

Here matrices \([\bar{A}], [C]\) are the corresponding FEM matrices of discrete model, explained in [5]; \(\bar{F_i} = \gamma_{\text{red}} \cdot F_i\) corresponds the vector of considered load combination \(i\).

The necessary residual values of analysis problem are obtained applying the vector of plastic multipliers \(\lambda_i\):

\[ S_r = [G] \sum_{i=1}^{\xi} [\Phi] \lambda_i, \quad (23) \]
\[ u_r = [H] \sum_{i=1}^{\xi} [\Phi] \lambda_i, \quad (24) \]
where matrices $[G]$ and $[H]$ are obtained by combining the primary matrices $[A]$ and $[D]$ [5].

5. Pedestrian bridge optimization and analysis

Consider carrying structure of pedestrian steel bridge, subjected by 4 load combinations, id est $i = 1, ..., 4$ (see Fig 1). Beam is connected with arch of shape conforming the relation $y = 12 - \frac{1}{75} x^2$ (Cartesian coordinate system is applied at left support; $x$ corresponds span variation of structure, $y$ corresponds arch height variation).

Discrete FEM model of structure contains 52 finite elements with total DOF = 98 (see Fig 2.) Note that not all directions are shown for clarity of a figure. Stress state of each cross-section $j$ is described by bending moment $M_j$ and axial force $N_j$, the total number of internal forces of the whole structure is 124. Load intensities are: permanent load $q_n = 24,36$ kN/m, live load is $q_k = 16,50$ kN/m. These distributed loads for considered load combinations are transferred to nodal concentrated loads.

Structural members are assigned to 6 different types of IPE type profiles, id est the total number of optimized limit bending moments is 6. Yield limit of steel is 235 MPa, density 7850 kg/m$^3$. A dynamics of optimization process is given in Table 1. Here the row 0 corresponds the limit bending moments, compatible with above given starting areas of structural members. The last row corresponds the actual cross-sectional areas and the total mass of the optimum structure. Limit bending moments are given in MPa. Note that problem contains per 2412 constraints in terms of equalities and inequalities.

The response of above obtained optimal structure (id est corresponding the limit equilibrium problem solution $M_{0}^{\text{min}}$) was investigated by means of structural analysis problem (21)-(22) to identify upper and lower available displacement variation bounds $u_{\text{adm}}$ and $u_{\text{adm}}$, those being possibly closer the structure state at plastic collapse. Thus, the considered load combinations were multiplied by reduction factor $\gamma_{\text{red}} = 0,996$ (for loads $\tilde{q}_n = \gamma_{\text{red}} \cdot q_n$ and $\tilde{q}_k = \gamma_{\text{red}} \cdot q_k$).

Maximal displacement (deflection) of 24.55 cm magnitude developed for the second load combination at beam node-connection with the second column from left. Other linear displacements were less. Plastic deformations appeared in 17 cross-sections due to all load combinations. The displacement variation bounds, determined in this way, ensure the successful solution of the full optimization problem, resulting an obtaining the optimal elastoplastic structure, responding to load combinations within prescribed displacement variation bounds.

The original software on the basis of standard computational mathematics package “Matlab” was created and tested for realizing of the developed mathematical models and algorithms for limit equilibrium optimization and for the elastoplastic response analysis problems of bridge structure.

Solution of limit equilibrium optimization problem

Iterative optimization problem (17)-(20) solution (id est the obtaining the lower bounds limit bending moments $M_{0}^{\text{min}}$ for problem (1)-(7)). Problem convergence was reached per 15 optimization loops. For starting point the members were taken of following areas (in cm$^2$): $A = [100; 100; 150; 150; 50; 50]^T$.

Analysis problem solution

![Fig 1. Design scheme of carrying structure of pedestrian steel bridge with optimized structural members and load combinations](image)
Fig 2. Discrete FEM model of structure

<table>
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<th>Optimization loop</th>
<th>$M_{01}$</th>
<th>$M_{02}$</th>
<th>$M_{03}$</th>
<th>$M_{04}$</th>
<th>$M_{05}$</th>
<th>$M_{06}$</th>
<th>Objective function (17) magnitude</th>
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By slightly increasing reduction factor magnitude the optimal structure was analyzed (via solving corresponding analysis problems) aiming to identify the loading limit, until the structure responses load combinations only in elastic range. The limit was found to correspond $\hat{\gamma} = 0.582$. The extreme structural displacement (deflection) of 5,039 cm corresponds the first load combination. Note that the magnitude of extreme displacement is much smaller comparing the any extreme displacement of the structure, obtained when allowing plastic deforming. This result clearly indicates that the available strength resource is not employed in case of elastic optimization/design of structure for admitted by design codes displacement magnitudes.

6. Conclusions

1. The presented limit equilibrium optimization and analysis techniques for complex structures serve a reliable basement for full optimization problem solution (determining the starting point of iterative solution procedures, defining reliable constraints for minimum design values, etc) and structure behaviour analysis at critical response states.

2. The developed mathematical models, algorithms and computational programs for limit equilibrium optimization and analysis of steel bridge structure illustrated a high efficiency when applied for large structures. The optimization conditions and considered structure were adapted to real design: evaluation of load combinations; problem constraints were compatible with codified requirements; set of standard steel profiles was employed; common action of bending moment and axial force has been evaluated.

3. Elastoplastic analysis of structural response viewed that elastic displacements are much smaller the admitted by codes ones. Therefore the essential strength/material resource of the steel bridge is not employed when allowing only elastic deforming.

The subsequent development of the full optimization (including stiffness constraints) problem formulation, a creation of algorithm and numerical solution is in fast progress: first reliable results are already obtained for considered pedestrian steel bridge optimization. The above presented results of limit equilibrium optimization and elastoplastic analysis were completely employed in further investigation of considered pedestrian steel bridge (solution of full structural optimization problem).

References


