RESEARCH OF FRICTIONAL DAMPER APPLICATION FOR OSCILLATIONS SUPPRESSION PROCESS IN NONLINEAR SYSTEMS

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Abstract. The model of a frictional damper was applied for suppression of oscillations in the nonlinear mechanical systems. Applying dampers in nonlinear systems it is very important to determine the values of system parameters those let the dampers operate most effectively. From the investigation of nonlinear system it has been determined that a frictional dampers frequency diapason in nonlinear system is wider than that in the linear case.

Keywords: frictional damper, harmonic linearization, nonlinear system, oscillations.

1. Introduction

One of the efficient approaches to fight the harmful vibrations is the application of dynamic dampers for mechanical systems. Dynamic dampers are advantageous because in application the main construction remains unchanged. If the force affecting the main mass causes dangerously large oscillation amplitudes, then by connecting the dynamic damper, i.e. additional mass, it is possible to decrease the main mass amplitude significantly [2]. When dynamic damper without frictions is applied, additional mass enforces the second frequency resonance and the damper remains efficient in row frequency diapason only. Hereby, dynamic damper without frictions is applied only in cases when excitation frequencies are stabilе. In other frequencies diapasons such damper can increase oscillations amplitudes significantly. For these reasons it can not be applied in the cases when harmonic excitations frequency is changing or when excitation is periodical and consists of several harmoniums as the damper can be regulated only for one harmonium oscillations suppression. Any frequencies diapasons, firstly, resonance zone oscillations, are efficiently suppressed applying frictional dynamic damper.

Analyzed effects can be found in manufacturing of precise devices, in rotational mechanisms kinematical pores with gaps, also in switchers, dent mechanisms.

2. Mathematical model development and solving analysis

Dynamic model is defined by system of two equations:

\[
\begin{align*}
mx + f(x) - c_1(x_1 - x) + h(x - x_1) &= A \sin \omega t, \\
m_1\ddot{x}_1 + c_1(x_1 - x) + h(x_1 - \dot{x}) &= 0
\end{align*}
\]

where: \( f(x) = \begin{cases} c(x + x_1), x > 0 \\ c(x - x_1), x < 0 \end{cases} \) .... \( (2) \)

here \( x, x_1 \) – coordinates measured on static balanced system position; \( m_1, c_1 \) – parameters of linear dynamic damper; \( m \) – main system mass; \( h \) – dampers’ frictions coefficient; \( c \) – main mass stiffness characteristic parameter.

Nonlinear stiffness characteristic becomes linear in applying this equation [2]:

\[
\begin{align*}
\tilde{f}(x) &= kx = \frac{4c_1v}{\pi B} + c \end{align*}
\]

here \( B \) – main mass amplitude.

Equations system can be transformed into the matrix form:

\[
M\ddot{q} + H\dot{q} + cq = p(t),
\]
here 
\[ M = \begin{pmatrix} m & 0 \\ 0 & m_i \end{pmatrix}, \quad H = \begin{pmatrix} h & -h \\ -h & h \end{pmatrix}, \quad C = \begin{pmatrix} k + c_i & -c_i \\ -c_i & -c_i \end{pmatrix} \]

\[ q = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} e^{i\omega t}, \quad p(t) = \begin{pmatrix} Ae^{i\omega t} \\ 0 \end{pmatrix} \]

The reaction to harmonic excitation is expressed in certain form:
\[ q = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} e^{i\omega t}, \quad \text{where} \]

here \( x, x_j \) – complex oscillations altitudes incorporated into matrix equation:
\[ C + i\omega H - \omega^2 M \]

Matrices' 
\[ z(\omega) = \left[ C + i\omega H - \omega^2 M \right] \]

\[ \text{reverse matrix} \quad H(\omega) = z^{-1}(\omega) \]

shows systems' displacement amplitudes influenced by excitation. Complex reaction amplitudes are calculated from the equation:
\[ \begin{pmatrix} k + c_i + i\omega h - \omega^2 m & -(c_i + i\omega h) \\ -(c_i + i\omega h) & c_i + i\omega h - \omega^2 m_i \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A \\ 0 \end{pmatrix} \quad (4) \]

\[ x_i = \frac{(c_i + i\omega h - \omega^2 m_i)A}{(k + c_i + i\omega h - \omega^2 m)(c_i + i\omega h - \omega^2 m_i) - (c_i + i\omega h)^2}, \]

\[ x_q = \frac{(c_i + i\omega h)A}{(k + c_i + i\omega h - \omega^2 m)(c_i + i\omega h - \omega^2 m_i) - (c_i + i\omega h)^2}. \quad (5) \]

After the module square is calculated, the real amplitude meaning is obtained:
\[ \left| \frac{x}{A} \right|^2 = \frac{(c_i - \omega^2 m_i)^2 + \omega^2 h^2}{((c_i - \omega^2 m_i)(k - \omega^2 m) - c_i \omega^2 m_i)^2 + \omega^2 h^2(k - \omega^2 m_i - \omega^2 m_i)} \]

This expression can be transformed into non dimensional form by denominations:
\[ \mu = \frac{m_i}{m}, \quad \omega_2 = \sqrt{\frac{c}{m}} - \text{self proportional additional mass}, \]

frequency of the main mass, \( \omega_1 = \sqrt{\frac{c_i}{m_i}} - \text{self frequency of the damper}, \]

\( \beta = \frac{h}{\omega_2} - \text{excitation frequency connected with resonance frequency of} \]

the main system, \( x_0 = \frac{A}{C} - \text{static movement of the main system,} \]

system, \( \alpha = \frac{\omega}{\omega_1} - \text{coefficient,} \quad h = \frac{h_i}{h_{i_0}}, \)

where
\[ h_{i_0} = 2\sqrt{c_i m_i = 2\omega_1 m_i}. \]

\[ \left| \frac{x}{A} \right|^2 = \frac{m_i\left(\left(\frac{c_i}{m_i} - \omega^2 + \omega^2 h^2\right)\right)}{m\left(\left(\frac{c_i}{m_i} - \omega^2 - c_i \omega^2 m_i\right)^2 + \omega^2 h^2(k - \omega^2 m_i - \omega^2 m_i)\right)} \]

\[ \left| \frac{x}{A} \right|^2 = \frac{m_i\left(\left(\frac{c_i}{m_i} - \omega^2 + \omega^2 h^2\right)\right)}{m\left(\left(\frac{c_i}{m_i} - \omega^2 - c_i \omega^2 m_i\right)^2 + \omega^2 h^2(k - \omega^2 m_i - \omega^2 m_i)\right)} \]

\[ \left(\frac{x}{A}\right)^2 = \frac{\left(\alpha^2 - \beta^2\right)}{\left(\alpha^2 - \beta^2\right)\left(\alpha^2 - \beta^2\right) - \alpha^2 \beta^2 \mu} \quad (7) \]

\[ \left(\frac{x}{A}\right)^2 = \frac{2h a}{\left(\alpha^2 - \beta^2\right)\left(\alpha^2 - \beta^2\right) - \alpha^2 \beta^2 \mu} \quad (8) \]

All the amplitude frequency characteristics cross points P and Q. When their ordinates are equal, the widest suppression diapason is acquired. When amplitude frequency characteristics are equalized coordinates of points P and Q are calculated. Nontrivial solution is obtained from the equation:
\[ \frac{1}{\left(\alpha^2 - \beta^2\right)\left(\alpha^2 - \beta^2\right) - \alpha^2 \beta^2 \mu} = \frac{\alpha^2 - \beta^2}{\alpha^2 - \beta^2\left(\frac{k}{c} - \beta^2\right) - \alpha^2 \beta^2 \mu} \]

\[ \beta^2(2 + \mu) - 2\beta^2\left(\frac{k}{c} + \alpha^2 \beta^2 + \frac{k}{c}\right) + \frac{\alpha^2 k}{c} = 0 \quad (9) \]

In solving the equation (9) abscises of points P and Q are evaluated. Differently than in linear case, abscises depend on the mass amplitude because linear transformed stiffness characteristics coefficient depends on amplitude \( k = k\left(\frac{x}{x_0}\right) \). Ordinates evaluated
when the limit is near to \( h \to \infty \) in amplitude frequency characteristic:

\[
\frac{x}{x_0} = \frac{1}{1 - \beta_1^2(1 + \mu)}, \quad \text{when } \beta_1 < 1
\]

\[
\frac{x}{x_0} = \frac{1}{\beta_2^2(1 + \mu) - 1}, \quad \text{when } \beta_2 > 1
\]

When system’s reaction is minimized, ordinates of points P and Q are equalized:

\[
\frac{1}{1 - \beta_1^2(1 + \mu)} = \frac{1}{\beta_2^2(1 + \mu) - 1}
\]

\[
\beta_1^2 + \beta_2^2 = \frac{2}{1 + \mu}. \quad (10)
\]

Basing on Vieta theorem, the equation is:

\[
\beta_1^2 + \beta_2^2 = \frac{\alpha^2(1 + \mu) + \frac{k}{c}}{1 + \frac{\mu}{2}}. \quad (11)
\]

When equations (10) and (11) are equalized, the equation from \( \alpha \) point is:

\[
\frac{2}{1 + \mu} = \frac{\alpha^2(1 + \mu) + \frac{k}{c}}{1 + \frac{\mu}{2}}, \quad \frac{\alpha^2}{c} = 1 + \frac{1}{1 + \mu} - \alpha^2(1 + \mu),
\]

\[
\alpha^2 = \frac{1}{1 + \mu} \left( \frac{\frac{k}{c} + 1}{1 + \mu} \right). \quad (12)
\]

Differently than in linear case, optimal frictional damper parameters selection depends on amplitude. By adding coefficient \( k \) expression from the equation (3) is:

\[
k = \frac{4x}{c} + \frac{4x_0}{x} + 1 = \frac{4}{x/x_0} + 1 = \frac{4}{x_0/x}. \quad (13)
\]

into equation (12), the equation of \( \alpha^2 \) is inserted:

\[
\alpha^2 = \frac{1}{1 + \mu} \left( 1 - \frac{4}{x/x_0} \right)^2 + 1 + \frac{1}{1 + \mu} \leq \frac{1}{(1 + \mu)^2}
\]

\[
\alpha \leq \frac{1}{1 + \mu} < 1.
\]

It can be realized that damper’s self frequency must be lower than the system’ self frequency:

\[
\alpha = \frac{\omega_1}{\omega_2} = \sqrt{\frac{c_1}{c\mu}} \leq \frac{1}{1 + \mu}.
\]

Accordingly, \( 0 < \frac{c_1}{c} \leq \frac{\mu}{(1 + \mu)^2} \).  \quad (14)

Frictional damper is optimally regulated when it is meeting inequality (14).

From minimizing system’s reaction, it can be achieved that fixed points P and Q ordinates will be evaluated by the same meaning, analogous to linear case. It obtained by inserting into frequencies equation (9) the equation (12). In the case when \( \alpha = \alpha_{opt} \), fixed points are located in the points with abscises equal to:

\[
\beta_1^2 - \frac{2\beta_1^2}{1 + \mu} - \frac{2}{(1 + \mu)^2} = 0.
\]

accordingly, \( \beta_1^2 = \frac{1}{1 + \mu} \left( 1 - \frac{\mu}{\mu + 2} \right) \), and the fixed points P and Q ordinates will be: \( x = \frac{\mu}{\mu + 2} \).

For the same system spring loaded damper can be used when excitation frequencies are steady. Differently than in frictional damper case the main mass amplitude can be close to zero but only in narrow frequencies diapason. The system with spring loaded damper can be described in differential equations system:

\[
\begin{aligned}
mx'' + c_1(x_1 - x) &= A \sin \omega t, \\
m_1x_1 + c_1(x_1 - x) &= 0
\end{aligned}
\]

(15)

here \( x, x_1 \) - coordinates measured in the systems static equilibrium position, \( m_1, c_1 \) - parameters of linear spring loaded damper, \( m \) – main system mass, \( f(x) \) - tightness characteristic (2).

Non linear tightness characteristic is straightened by applying harmonic linearization method. Systems solutions will be:

\[
\begin{aligned}
x &= B \sin \omega t, \quad \text{here } \omega t = \tau, \\
x_1 &= B_1 \sin \omega t
\end{aligned}
\]

(16)

Straightened tightness characteristic coefficient \( k \) is calculated:

\[
k(B) = \frac{1}{\pi B} \int_0^{2\pi} f(B \sin \tau) \cdot \sin \tau d\tau =
\]

\[
= \frac{4}{\pi B} \left( \int_0^{\pi} (cB \sin \tau + cx) \sin \tau d\tau \right) =
\]

\[
= \frac{4}{\pi B} \left( cB \int_0^{\pi} \left( l - \cos 2\pi \right) d\tau - cx \cdot \cos \tau \right) =
\]

\[
= \frac{4}{\pi B} \left( cB \frac{\pi}{4} + cx \right) = \frac{4cx}{\pi B} + C.
\]
By inserting into the system (15) solutions’ (16) expressions and straightening tightness characteristic, the equation system is transformed into the following system:

$$\begin{align*}
-m\omega^2 B + k B - c_i (B_i - B) &= A \\
-m\omega^2 B_i + c_i (B_i - B) &= 0
\end{align*}$$  \hspace{1cm} (17)

The amplitude of the dynamic damper is calculated from the second equation of the system (17):

$$B_i = \frac{c_i B}{c_i - m\omega^2} = \frac{\omega_1^2 B}{\omega_0^2 - \omega^2}$$

The main mass amplitude $B$, from inserting $B_i$ expression into the systems (17) first equation, will be:

$$B = \frac{c_1 B}{A} \cdot \frac{1}{\omega_0^2 \left(1 - \frac{k}{\mu} \left(1 - \frac{\omega_0^2}{\omega^2}\right) - \frac{1}{\omega_0^2}\right)}$$

here $\mu = \frac{m}{m_1}$, $\omega_1^2 = \frac{c_1}{m_1}$.

By transforming the above written equation, the amplitude frequency characteristic with dynamic damper is obtained:

$$B = \frac{c_1 B}{A} \cdot \frac{1 - \gamma^2}{\gamma^2 \left(1 - \frac{k}{\mu} \left(1 - \frac{\omega_1^2}{\gamma^2}ight) - \frac{1}{\omega_1^2}\right)}$$  \hspace{1cm} (18)

here $\gamma = \frac{\omega}{\omega_0}$.

From the (18) equation it can be noted that $B \rightarrow 0$, when $\gamma^2 \rightarrow 1$, i.e. choosing dynamic dampers parameters, when $\frac{c_1}{m_1} = \omega_1^2 \rightarrow \omega^2$ oscillations in the system are suppressed.

In non dimensional coordinates amplitude frequencies characteristic is expressed:

$$\frac{B}{A} = \frac{1 - \frac{2}{\mu} \left(1 + \frac{c}{c_1} - \frac{1}{\gamma^2} - 1\right)}{\gamma^2 \left(1 - \frac{k}{\mu} \left(1 - \frac{\gamma^2}{\gamma^2}ight) - \frac{1}{\gamma^2}\right) - c_1 x_x}$$

here $k = \frac{4c}{\pi c_1} x_x + c = \frac{B}{x_y} x_y$.

In non dimensional coordinates amplitude frequency characteristic without dynamic damper is calculated from the equation:

$$m\ddot{x} + kx = A \sin \omega t,$$

by inserting into it $x = B \sin \omega t$:

$$-m\omega^2 B + k B = A.$$

Accordingly:

$$\frac{B}{x_y} \left(\frac{k}{\gamma^2} - \gamma_1^2\right) = \frac{A}{c x_y}, \quad \text{here } \gamma_1 = \frac{\omega}{\omega_2}, \quad \omega_2^2 = \frac{c}{m}.$$  \hspace{1cm} (19)

Near the limit $\frac{x}{x_y} \rightarrow \infty$ from the amplitude - frequency characteristic, characteristic of an equation for frequencies calculation is obtained:

$$\frac{\gamma^2 + \frac{c}{c_1} - \gamma^2}{\mu} = 0$$  \hspace{1cm} (20)

By solving equation (20), the system will have resonance oscillations, when

$$\gamma_{c_2}^2 = \frac{-4m^2}{c^2} \left(1 + \mu \left(\frac{c}{c_1} + 1\right)^2\right) \frac{c}{c_1}.$$

Equations (20) solution existence conditions are satisfied because:

$$\left(1 + \mu \left(\frac{c}{c_1} + 1\right)^2\right) - 4\mu \geq 0.$$  \hspace{1cm} (21)

The distance between resonances $\Delta$ is equal to:

$$\Delta = \gamma_1^2 - \gamma_2^2 = \left(\frac{1}{1 - \mu \left(\frac{c}{c_1} + 1\right)^2} - 4\mu\right) \frac{c}{c_1}.$$  \hspace{1cm} (22)

In comparison with linear case, when the distance between resonances is equal to $\gamma_1^2 - \gamma_2^2 = \sqrt{4\mu + \mu^2}$ it can not be stated that damper works in wider frequency diapason. This effect can be achieved only by choosing parameters accordingly.

When condition $\left(1 - \mu \left(\frac{c}{c_1} + 1\right)^2\right)^2 > \mu^2$ will be satisfied, dynamic damper will work in wider frequencies diapason than in linear case:

$$\mu \geq \frac{c}{c_1} \quad \text{or} \quad 0 < \mu < \frac{c_1}{2c_1 + c}.$$
Knowing that \( \frac{m_1}{m} = \mu < 1 \), dampers parameters variation domain will be:

\[
0 < \mu < \frac{c}{2c_1 + c} < 1 \quad \text{or} \quad \mu > \frac{c}{c_1}.
\]

When dampers parameters \( m_1, c_1 \) are satisfying the inequality \( \frac{c_1}{c} < \frac{m_1}{m} < \frac{c_1}{2c_1 + c} \), dynamic damper in non linear system is working in a more narrow frequencies diapason than in linear system. The main mass amplitude is minimized in wider frequencies diapason than in the linear case when dampers parameters are satisfying following conditions:

\[
\frac{c_1}{c} < \frac{m_1}{m} < \frac{c_1}{2c_1 + c}
\]

Frictional dynamic damper and spring – loaded dynamic damper, when elasticity force is equal (2), amplitude frequencies comparison is showed in Figure 1 and Figure 5.

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**Fig 1.** Dynamic damper amplitude frequency characteristics, when \( A=0.2, \ c=0.8, \ c_1=0.04, \ \mu=0.4 \)

**Fig 2.** Dynamic frictional damper amplitude frequency characteristics, when \( A=0.6, \ c=0.2, \ h=0.1, \ \mu=0.4 \)

**Fig 3.** Dynamic frictional damper amplitude frequency characteristics, when \( A=0.4, \ c=0.4, \ c_1=0.04, \ \mu=0.3 \)

**Fig 4.** Dynamic damper amplitude frequency characteristics, when \( A=0.4, \ c=0.4, \ c_1=0.04, \ \mu=0.2 \)

**Fig 5.** Dynamic frictional damper amplitude frequency characteristics, when \( A=0.7, \ \mu=0.5, \ h=0.2, \ c=0.2 \)
3. Conclusions

1. Optimal frictional damper parameters selection depends on the main mass amplitude.
2. Applying frictional damper in nonlinear system its reaction can be minimized analogous as in linear system.
3. Frictional dynamic dampers excitation frequency diapason is wider than the spring-loaded one.
4. Nonlinear systems with adjustments research revealed that in the non linear system dynamic dampers frequency diapason is not always wider than in the linear case.
5. Nonlinear system parameters variation domains were found where dynamic damper is more efficient than in linear system.

References