THE NUMERICAL AND ANALYTICAL METHODS OF CALCULATIONS OF TWO-DIMENSIONAL TEMPERATURE FIELDS IN DANGEROUS MEMBERS OF BUILDING ENCLOSURES

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Abstract. The object of research: two-dimensional temperature fields in members of building enclosures, most dangerous from the point of view of a capability of a freezing or vapor condensing in the cold season of year, including in an outside corner and on joints of internal and outside walls and overlaps. The goal of research: the engineering formulas for definition of temperature in stagnation points and cross-sections of the reviewed members and the recommendations for using of these formulas at an estimation of conformity of enclosures to the requirements of thermal safety. The methods applied: the modern numerical and analytical methods of calculations of two-dimensional temperature fields with the experimental endorsement of offered relations for an outside corner of a building by results of full-scale measurements of temperature. The investigation findings: the new regularities of temperature behavior in dangerous members of modern building enclosures and the new values of corresponding numerical coefficients.

1. Introduction

Recently, in connection with occurrence and distribution of outside envelopes designs with heightened resistance to heat transfer and with availability of layers from effective heat-insulating stuffs, we newly arise problems tangent definitions of temperatures on an internal surface of reference members of such protections. In particular, it falls into to outside corners, window acclivities, joints and other segments with a two temperature field. Such necessity of calculus in the maiden sequential queue is connected to safety control of operation of buildings, as just in listed members the condensation of water vapors, обмерзание and other unfavorable phenomena is possible. Especially it is actual in connection with acceptance of the Law of Russian Federation "About technical regulation", which one the main attention gives just to regulation of safety issues.

2. Methodology of research

Generally it is necessary to apply to calculation of two temperature fields the numerical methods. As a rule, the method of grids, founded on finite-difference approximating of a differential heat conduction equation (Laplace) [1] will be used. Let's consider, that thus is

received for such often of meeting and relevant member of designs, as an outside corner. For determining yardstick here it is expedient to select dimensionless relation R_{in}/R_o of resistance to heat exchange on an internal surface of a corner $R_{in} = 1/\alpha_{in}$, (m[2]·K)/W, to full resistance of a wall to heat transfer far from a corner R_o in the same dimension. Here α_{in} is a factor of full heat exchange on an internal surface of a wall, W/(m[2]·K). As instituted the conditional relative thermal resistance r', with the help which one temperature directly in a corner was accepted τ_c it is possible to compute under the formula:

$$\tau_c = t_{in} - r'(t_{in} - t_{ex}) \qquad (1)$$

Here t_{in} and t_{ex} are accordingly temperature of internal and external air, °C. It is clear, that far from a corner $r' = R_{in}/R_o$, and for τ_c necessarily there should be $r' > R_{in}/R_o$.

The outcomes of numerical calculations on definition r' are well approximated by following expression, fair in a broad band R_o down to 8 (m[2]·K)/W:

$$r' = A\left(\frac{R_{in}}{R_o}\right)^{2/3} \qquad (2)$$

Here $A = 1$ for a single-layer wall and $A = 0.75$ for multilayer with a design heat-conducting layer arranged

from the interior of a wall. For the first time formula (2) was offered in activity [2]. But it is possible also to record equivalent on accuracy a ratio for the first time published in [3]:

$$r' = B\left(\frac{R_{in}}{R_o} + \sqrt{\frac{R_{in}}{2R_o}}\right) \qquad (2a)$$

In this case $B = \frac{2}{3}$ for a single-layer wall and $B = \frac{1}{2}$ for multilayer wall. Certainly, the expression (2a) on a kind a little is more difficult, but from the point of view of a simplicity of calculus, probably, can appear preferential, as does not demand exponentiation in a fractional degree which is distinct from, that is why is accessible to calculations on the elementary microcalculator.

Basically frame of the formula (2a) can be received and theoretically – direct solution of an equation of the Laplace with a boundary condition of the 1-st kind. In case of an outside corner it does not introduce insuperable complexities. Namely, passing to dimensionless parameters and considering depth of a corner equal 1, and also setting temperature on an outside surface, equal 0, and on internal – accordingly 1, we receive a general structure of the solution in a complex kind [4]:

$$t' = a_1 z^2 + a_2 \sqrt{1 + i - z} + a_3 (1 + i - z)^{3/2} + C \qquad (3)$$

Here $z = (x + iy)/\delta$ is complex relative coordinate, and δ is the wall thickness, m; t' is complex dimensionless temperature. For a beginning of coordinates the outside fastigium of a corner is received. The maiden and second addends respond a potential distribution accordingly for angles $\pi/2$ and $3\pi/2$, i.e. near to an outside and internal surface, third occurs for the count of symmetry of a problem and responds the angle $3\pi/4$ between an internal surface and symmetry axis. Constants a1 … a3 and C are received by permutation of boundary conditions, allowing, that at $z = 0$ $t' = 0$ and $\partial t'/\partial z = 0$, and at $z = 1$ $t' = 1$ and $\partial t'/\partial z \to \infty$. The conditions on derivative follow from [3]. We are interested in temperature on a symmetry axis, i.e. bisectrix of a corner, where $x = y$. Then on sense of a problem $R_{in}/R_o = 1 - Re(z)$, and value r' is not that diverse, as $1 - Re(t')$, where $Re(z)$ and $Re(t')$ are real parts of z and t' correspondingly. At $x = y$ we have $Re(z) = x$. Then in real area $a_1 = C = \frac{1}{2}$, $a_2 = -1/\sqrt{2}$, $a_3 = (1/\sqrt{2} - 1/2)$, also the formula for r ' is discovered:

$$r' = 1 - Re(t') = \frac{R_x}{R_o} - \frac{1}{2}\left(\frac{R_x}{R_o}\right)^2 + \sqrt{\frac{R_x}{2R_o}} - \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)\left(\frac{R_x}{R_o}\right)^{3/2} \quad (4)$$

Here R_x is the current thermal resistance from internal air to a considered point. Apparently, at replacement of a boundary conditions of the 3-rd kind by conditions of the 1-st kind for calculus of temperature on an internal surface in quality R_x it is necessary to use R_{in}. At $R_{in}/R_o < 0.15$, that there corresponds to $R_o > 1$ (m²·K)/W, second and fourth addend in (4) can be

neglected. Error will be lower than 0.023, i.e. less than 5 %, and in the party of uprating r', or underestimation of temperature in a corner, that gives a reserve. Then we receive relation conterminous with (2a) to within a numeric factor.

Factor $\frac{2}{3}$ in the formula (2a) is received at transition to boundary conditions of the 3-rd kind and is connected to change of nature of heat exchange on an internal surface. He is determined by matching of outcomes of numerical calculations and on expression (4). In this case relation (2) is approximating on relation to (2a), and the known formula [5] for calculus of an air flow infiltrating through windows (also with an exponent falls into with her the same as $\frac{2}{3}$ at a difference of pressure) to expression reflecting physics of process of an infiltration (a speed key laminar and turbulent) and keeping the laminar of an and of turbulent in degrees 1 and $\frac{1}{2}$. Then it is possible to consider, what the factor in (2a) is equal to $\frac{2}{3}$ just because of availability of a speed key of degrees 1 and $\frac{1}{2}$ for R_{in}/R_o.

On Fig.1 the dimensionless profiles of temperature are shown: by a solid heavy line – under the data of numerical calculations at a boundary condition of the 1-st kind, and for matching by a heavy line with daggers – under the formula (4). It is visible, that the divergence does not exceed 0.02, i.e. the relation (4) correctly describes a temperature field on an axis of a corner. The light lines routine outcomes of numerical calculations at a boundary conditions of the 3-rd kind and different ratio R_{in}/R_o. It is easy to note, that relative temperature on an internal surface of a corner, equal ordinate for the left-hand test leads of the figured curves, really is higher, than under conditions of the 1-st kind. And if to conduct an envelope through a point (0; 1) and through these test leads, she actually will correspond to expressions (2) – (2a), i.e. with a diminished numerical constant.

It is necessary also to note, that the expression for r' in the form (2) can be received and method of conformal transformations [4]. The business that in close proximity from an internal surface of an outside corner is possible as a first approximation to neglect availability of an outside surface, i.e. to consider depth of a wall infinite, and massive of a corner not restricted from an exterior. Then there is the angle $3\pi/2$, which one can be received from a upper half plane, i.e. angle, equal to π, by transformation z' = z^{2/3}, where $2/3 = \pi/(3\pi/2)$. In this case for a beginning of coordinates internal fastigium of a corner is received, i.e. joint of walls on the side of the room. Therefore now $r' = Re(z')$, whence formula (2) directly follows. Or else, the relation in the form (2) represents a limiting case of general expression (4) at $R_{in}/R_o \to 0$, or at large values R_o. However, as against (2a), it is extended for an angle of any size: In this case instead of 2/3 as an exponent it is necessary to substitute π/φ, where φ is radian measure of the angle. As R_{in}/R_o always are less than 1, at $\varphi > \pi$ we have $r' > R_{in}/R_o$, i.e. for corners such as outside, versed in the room by the concave party, temperature on the joint will be lower, than on an internal surface of a wall far from a corner. To the contrary, at $\varphi < \pi$ we have $r' <$

R_{in}/R_o, i.e. for corners pressing in the room temperature on the joint will be heightened.

Experimental check of the formulas (2) and (2a) and the confrontation them with the available idealized data is more friend to conduct by usage of padding conditional relative thermal resistance in a corner r''. It demonstrates a relative decrease of temperature in a corner as contrasted to in temperature of an internal surface of a wall τ_{in} far from a corner. Pursuant to such definition $r'' = (\tau_{in} - \tau_c)/(t_{in} - t_e)$. For r'' in [6] the following relation, fair, however, only up to $R_o < 2.5$ (m²·K)/W is offered:

$$r'' = 0.18 - 0.042 \cdot R_o \qquad (5)$$

If we use expressions (2) or (2a), r'' it is possible to receive as $r' - R_{in}/R_o$.

Values r'', computed on the data of researches [7], [8], and also including of one of the writers of the present article engineer K.I. Lushin conducted in January – March, 2004 with the help of full-scale measurements in two-section apartment house in Moscow with 11 stages constructed on technology "Plastbau" with application of fixed formwork are resulted below. Such formwork hereinafter plays a role of a heat-insulating layer in outside enclosures. The sensing were made with the help of sensors by the way of thermoelectric couples, immured in an internal design layer of a wall directly for a surface. The signals from sensors acted in a computer established in center of supervision, where were converted to values of temperatures in points of the installation of sensors. The data outcomes for the first time were published in activity [9].

3. Results of the investigation

At processing of outcomes of sensing the resistance to heat transfer of an outside wall Ro in a zone of the joint was received equal 3.47 (m²·K)/W as mean between resistance of facade (3.65) and end (3.28) walls [7], and temperature – under the data of one of the co-authors of the present activity engineer K.I. Lushin, and also by [7, 8]. The factors of heat convection far from a corner and directly in a corner were taken with usage of the guidelines [10] at a level accordingly 1.2 and 1.55 W/(m²·K) depending on a difference of temperatures of air and on a surface. The factor of radiant heat exchange far from a corner) was adopted equal 5 W/(m²·K) with allowance for of optical behavior of an internal surface of a wall [6, 10], and in a corner – about 2.7 W/(m²·K), as the factor of an exposure in a zone of the joint only by little is more 0.5 [6]. Therefore, the factors of full heat exchange indispensable for calculation of the value R_{in} in expressions (2) and (2a), will appear at a level 6.2 and 4.15 W/(m²·K).

At further calculus the following outcomes were obtained:

– Under the formula (3) $r'' = 0.0343$;

– Under the simplified formula for a multilayer wall (2) $r'' = 0.0783$;

– That under the full formula (2a) $r'' = 0.0795$;

– On Fig.2 (approximating of experimental data) $r''_m = 0.0743$.

Correlation coefficient of sampling $r = 0.994$, mean quadratic deviation $\sigma = \pm 0.234$ °C, that does not exceed instrumental error, equal ± 0.25 °C, therefore it is possible to consider the obtained relation authentic. Besides the sum r''_m and relation σ to a mean difference $(t_{in} - t_e)_m = 27.35$ °C, giving to us the upper possible limit r''_{max} with allowance for of spread of experimental points, is peer 0.0828. Thus, the r'' values, calculated on the formulas, do not exceed value r''_{max}, therefore divergence of idealized and experimental outcomes is not statistically significant.
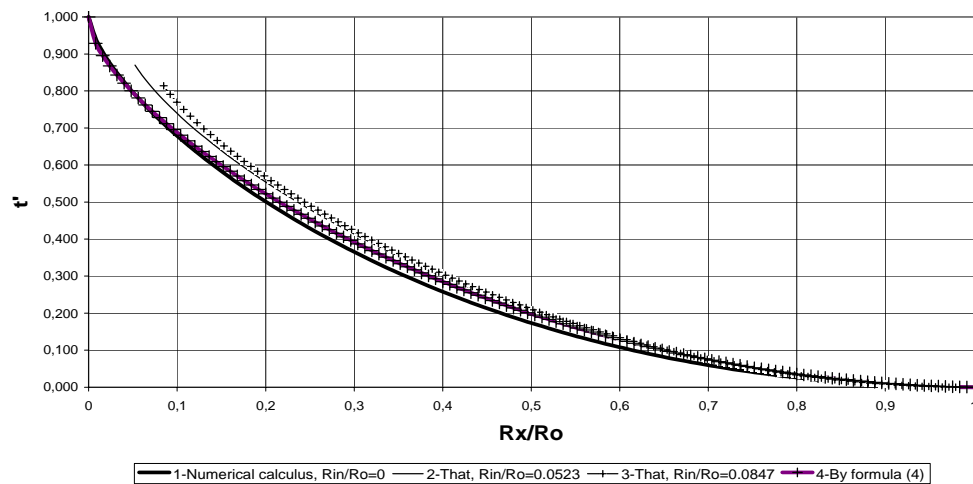

Fig. 1. Change of dimensionless temperature t' on angle bissectrix from ratio R_x/R_o.
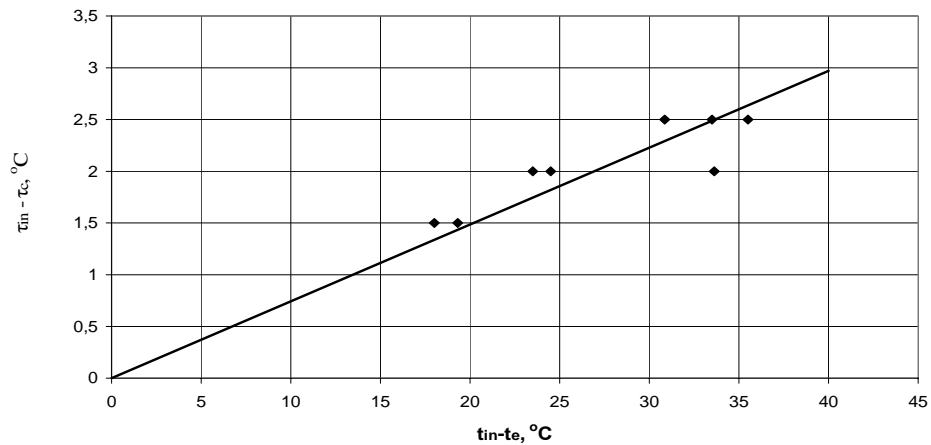
Fig. 2. Correlation dependence of difference $\tau_{in} - \tau_c$ from difference $t_{in} - t_e$.

In this paper the problem of distribution of temperature in an outside wall and wall sectioning two rooms with miscellaneous temperatures is reviewed also. The two-dimensional model of distribution of temperature was decided with the help of the classic scheme of calculation of heat flows with splitting of zones into a square grid. Thus the calculation was conducted for two segments with different number of members: 1) zones of an external wall of constant depth and 2) zones of a partition between internal rooms with set values of temperatures. For check of independence and convergence of the solution the calculations were conducted at different sequence of calculus of temperature of zones. The outcomes of calculations have shown full absence of any relation of distribution of temperature from the mentioned above order. The program was drawn up in medium of graphic programming LABVIEW. The outcomes of calculus were transmitted by the way of text file in the program MATCOP, permitting to receive distribution of temperature by the way of isotherms demonstrating jumps introduced on Fig.3. On a known temperature field it is easy to define heat flows transmitted through different configuration items.
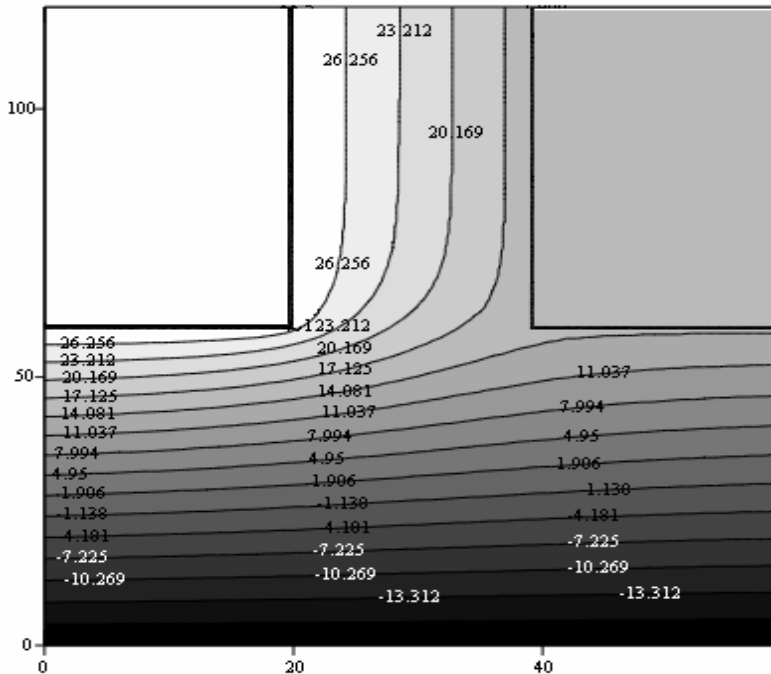
Fig.3. Distribution of temperature on cross-section of the joint of an outside and internal wall between rooms with different temperature of internal air (values on the axes show space dimensions of the wall in centimeters)

4. General conclusions

Therefore, the formulas (2) – (2a) for calculation r' by the data of available measurements are actually confirmed by experiment. Simultaneously it appears, that for the best concurrence with experiment the factors of internal heat exchange really are necessary for receiving in a corner – at a level 4 … 4.3 W/(m²·K). Besides it is necessary to substitute used values R_o which corresponds to segments of a wall far from heat-conducting actuations, therefore in expressions (2) – (2a) just them, instead of reduced resistance. The case

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can be special only, when the heat-conducting actuation places directly in a corner, for example, for a soaking masonry. Then in quality R_o it is necessary to apply resistance computed on actuation.

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