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Made in Tallinn, Estonia, 01.11.2011 by Ülo Hunt

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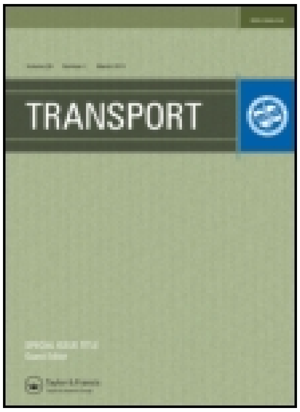

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MODELLING OF INTERNAL COMBUSTION ENGINES' EMISSION THROUGH THE USE OF TRAFFIC FLOW MATHEMATICAL MODELS

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Abstract. Road traffic flows on a straight road segment are modelled in this article. The mathematical model of traffic flows has been constructed by using the method of lumped parameters. CO_2, CO, CH, NO_x, PM regression equations of internal combustion engines' (ICE) emission has been developed. The accuracy of regression equations is 0.98÷0.99. The article presents assumptions for constructing the mathematical model, description of the mathematical model and gives simulation results. Traffic flow parameters, such as traffic flow concentration and traffic flow speed are presented as modelling results. ICE emission depending on the concentration and traffic flow speed are presented as well.

Keywords: traffic flow, mathematical modelling, internal combustion engine, emission, numerical simulation, fuel consumption.

1. Introduction

Road transport is one of the main inland transport modes providing house to house services for the people all over the world. Each inland territory is criss-crossed by inter-urban road and street network. Vehicle flows carry people, distribute, industrial freight and work equipment on these network elements. Majority of these road vehicles are driven by internal combustion engines; therefore besides practical use they also create a lot of problems, such as air pollution with combustion products and particulate metter, noise, vibration, utilization of used oil and other materials, recycling of cars and their parts. Cars consume a lot of energy; therefore when solving problems caused by them a lot of engineers and scientists put a lot of efforts to solve a wide range of problems starting from vehicle manufacturing to their utilization.

A lot of burning problems arise when cars are used and here the main problems to be solved by both engineers and scientists are pollution reduction and energy saving. When saving energy, cars become more environment-friendly. Various problems caused by vehicles are discussed in the article written by Makaras *et al*. (2011). The authors discuss vehicle dynamics in the flow, fuel consumption, the impact of environment on the dynamics of cars, touch upon the driver's model and various driving styles. Wang *et al*. (2008) presented various methods of fuel consumption and engines' emission measuring as well as coefficients of efficiency. Tanczos and Torok (2007) investigated climate fluctuation changes and energy consumption in Hungary. The article presents the dynamics of climate and CO_2 change as well as energy consumption in the Republic of Hungary. CO_2 emissions are presented as well: 1 mol of diesel (198 g) yields 14 mol or 616 g CO_2 and 1 mol (114 g) of petrol yields 8 mol or 352 g CO_2. Janulevičius *et al*. (2010) presented the methodology of determining energy consumption taking into account engine's capacity and specific fuel consumption. Wu and Liu (2011) in their article presented the methodology of calculating fuel consumption by taking into account such criteria as aerodynamic and rolling resistance. Fuel consumption model was constructed and based on the neural network theory. Smit *et al*. (2008) presented and generalized three emission models, where the impact of congestions on motor vehicles' emission is evaluated differently and present indicators to identify transport congestions. The article also presents congestion identification models. Jović and Đorić (2010) used programming package PTV Visio to model traffic flows on the urban street network and based on that present vehicle emissions. Jakimavičius and Burinskienė (2010) investigated vehicle flow optimization methods and their application possibilities when informing traffic users about the situation in the city, and in their article (Jakimavičius, Burinskienė 2009a, 2009b) presented the rating system of the Vilnius city residential areas by using expert methods and pointed out the residential areas with the highest traffic volumes of the city. Kliukas *et al.* (2008) investigated the impact of vibrations caused by vehicles on buildings with the aim to preserve cultural values in the city of Vilnius. Frequencies 1.3 and 10.8 Hz are presented as dangerous. Žiliūtė *et al.* (2010) investigated traffic flows and presented the data obtained on high intensity streets of Vilnius city. Sivilevičius (2011) investigates the interaction of transport system elements by taking into account traffic flow elements.

This article gives an example of applying mathematical models of traffic flows to simulate internal combustion engine (ICE) emission. The article does not investigate combustion reactions of ICE; however, it uses ICE emission dependences on the vehicle's movement speed, which are developed by using the values of emissions.

2. Assumptions of Constructing a Road Segment

When describing traffic flows, a traffic lane is used as a keyword. An assumption is taken that cars cannot drive on an opposite traffic lane; therefore, the road is split into separate traffic lanes and two-way roads are described in the mathematical model as a separate one-way road with one or several traffic lanes. In this model a traffic lane segment is taken as a finite-length line, which is divided into equal length segments the length of which is L (Fig. 1). The parameters of a road segment (traffic flow speed, traffic flow concentration or traffic flow intensity) accumulate at the end points. The point which connects two adjacent segments has a point common to both segments. The first and the last points of a traffic lane under investigation are data entry points at boundary conditions. Minimal and maximal values of traffic flow parameters for each road segment are set separately and may be different. Minimal values of traffic flow speed and concentration are usually equal to zero. Maximal possible value of speed is based on observations. Maximal permitted traffic flow concentration value may be calculated according to the following formula:

$$k = \frac{n_{veh}}{L_{i,i+1}}, \tag{1}$$

where: n_{veh} – a number of vehicles in segment (units), $L_{i,i+1}$ – a road segment length (m).

Let us suppose that a road segment cannot be overfilled. If a number of vehicles on a road segment reaches

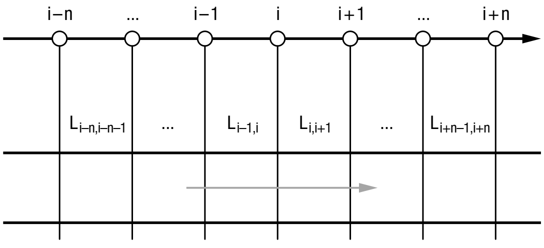

Fig. 1. Transport flow model on a straight segment of a traffic lane

an upper concentration limit, more vehicles cannot enter this road segment. If a road segment is full, the value of vehicles' flow remains the same, increases or slightly decreases, but is insufficient and the road segment ahead remains overfilled, vehicles start accumulating on the road segment subsequent to the full road segment.

Vehicle flow speed is limited as well. Each road segment has its own speed limit which can differ.

If the road has several one-way traffic lanes, an assumption that there is one traffic lane, the concentration parameters of which are proportionally increased, is taken in the simulation. This assumption enables to connect several one-way traffic lanes into one and the model becomes simpler because the migration of vehicles from one traffic lane to another does not have to be taken into account.

Traffic lane segments that are split in the model are numbered $L_{i-1,i}$, $L_{i+1,i}$ when $i = 1...n$, i – road point number. (Fig. 2). Points at the ends of this segment are merged with the ends of adjacent elements. The first point of an element is equal to the last point of the preceding element, and the last point of an element is equal to the first point of the subsequent element. Boundary conditions of the task are entered on boundary elements, the first point of the first element and the last point of the last road element.

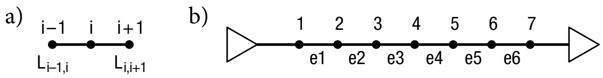

Fig. 2. Description of elements of a straight road: a – two elements $L_{i-1,i}$ and $L_{i+1,i}$, are connected at point i; b – a straight road segment drawn from several road segments connected at points 1÷7

Such points of boundary conditions (Fig. 2) are 1 and 7. At these points during the whole modelling time the input and output system information is known. Therefore, the value of the flow, which may be constant or change according to the law known in advance during the whole modelling time, is frequently entered into the system.

3. The Mathematical Model for Description of Traffic Flows and Internal Combustion Engines' (ICE) Emission

3.1. Mathematical Model of Traffic Flows

Equations describing traffic flows and the explanation of equation members are presented in the paper of Junevičius, Bogdevičius (2009). Hereinafter mathematical expressions are presented.

Mathematical expressions of traffic flow parameters are described by the following formulae:

$$\dot{k}_i = p_{in,i}(t) \cdot r_{k,in.,i} \cdot \left(1 - \frac{k_i(t)}{k_{\max,i}}\right) \cdot \left(\frac{q_{i-1}(t - \tau_{i-1})}{q_{\max,i-1}}\right) \cdot k_i(t) -$$

$$p_{out,i}(t) \cdot r_{k,out,i} \cdot \left(1 - \frac{k_{i+1}(t)}{k_{\max,i+1}}\right) \cdot \left(\frac{q_i(t)}{q_{\max,i}}\right) \cdot k_i(t); \tag{2}$$

$$\dot{v}_i = p_{in,i}(t) \cdot r_{v,in.,i} \cdot \left(\frac{v_{i-1}\left(t - \tau_{i-1,i}\right)}{L_{i-1,i}} \right) \cdot \left(1 - \frac{k_i(t)}{k_{\max,i}} \right) \cdot v_i(t) +$$
$$f_i\left(k_{i+1,i}\right) - p_{out,i}(t) \cdot r_{v,out} \cdot \left(\frac{1}{2} \frac{v_i(t) + v_{i+1}(t)}{L_{i+1,i}} \right) \times$$
$$\left(1 - \frac{k_{i+1}(t)}{k_{\max,i+1}} \right)^{m_1} \cdot v_i(t) - \left(\frac{v_i(t)}{v_{\max,i}} \right) \cdot e^{\left[\gamma_3 \left(\frac{k_i(t)}{k_{\max,i}} \right)^{m_2} \right] \cdot \left(\frac{v_i(t)}{v_{\max,i}} \right)}, \tag{3}$$

where (for simplicity parameters are described without indexes $i-1$, i, $i+1$): $k(t)$, $v(t)$ – sought traffic flow parameters: traffic flow concentration and speed; $k_{\max}$ – maximum possible flow concentration on a road segment (veh/m); $v_{\max}$ – maximum possible traffic flow speed on the element (m/s); L – discrete road segment length (m); τ – time interval necessary for a traffic flow to cover a road segment, the length of which is equal to L (s); m_1, m_2, γ – constants; q – traffic flow intensity (veh/s); $q_{\max}$ – maximum possible intensity of the flow (veh/s); $r_{k,in}$, $r_{k,out}$ – values of function introducing the correction for flow concentration change expression; $r_{v,in}$, $r_{v,out}$ – function values introducing correction for flow speed change expression (functions $r_{k,in}$, $r_{k,out}$ $r_{v,in}$, $r_{v,out}$ depend on the concentration and speed values and these functions may have different values when the flow enters and leaves the element); $p_{in}(t)$, $p_{out}(t)$ – probability of the flow entering the segment ahead (function $p_{in}(t)$, $p_{out}(t)$ values change in the course of time depending on the concentration and flow speed values' change or may be introduced as handling functions; $f_i(k_{i+1,i})$ the function which depends on the unknown parameter k (this function describes the condition of the road segment element preceding road point '*i*', i.e. it shows how intensively loaded this traffic lane segment is):

$$f_i = \begin{cases} e^{\gamma_2 \left(1 - \frac{\varepsilon_{i+1}}{\varepsilon_i} \right) \cdot \varepsilon_i \cdot \mathrm{sign}\left(p_{out,i}(t) \right) \cdot \mathrm{sign}\left(1 - \frac{\varepsilon_{i+1}}{\varepsilon_i} \right)}, & \text{if } \varepsilon_i > \varepsilon_{i+1} \text{ and } \varepsilon_i > 0; \\ 0, & \text{in other case;} \end{cases} \tag{4}$$

$$\varepsilon_i = \frac{k_i}{k_{\max,i}}. \tag{5}$$

The functions obtained when flow parameters change have the following advantages:

- both functions of parameters change in the course of time and influence each other.
- specified empirical functions $r_{k,in}$, $r_{k,out}$, $r_{v,in}$, $r_{v,out}$, which reduce sharp leaps of unknown parameters' values are introduced.
- functions allow the concentration value to obtain 0 value and functions do not approximate to ∞.

Members of the equation system (2) and (3) correct the sought parameters of the flow speed and concentration.

Equation (2) member $1 - \frac{k_i(t)}{k_{\max,i}}$ describes traffic flow concentration change speed at point '*i*'. Equation (2) member $1 - \frac{k_{i+1}(t)}{k_{\max,i+1}}$ describes the speed of traffic flow concentration change at point '*i*+1', and when traffic flow concentration on a traffic lane segment ahead of point '*i*', in the direction of traffic flow movement, the value of this member approximates to zero (element's loading approximates to the maximum possible concentration on the segment). Therefore, a restriction which does not allow to overfill a traffic lane segment element is established. Moreover, when traffic flow concentration at points '*i*' and '*i*+1' approximates to value $k_{\max}$, concentration change speed decreases.

Equation (2) member $\frac{q_{i-1}\left(t - \tau_{i-1}\right)}{q_{\max,i-1}}$ describes the condition of traffic flow on the segment preceding point '*i*' in the direction of traffic flow movement. This member describes the flow concentration increase or decrease at point '*i*' depending on the flow intensity on a traffic lane segment which precedes point '*i*'. Time interval τ_{i-1} during which the flow travels from one point to another is taken into account as well. This delay enables to describe the movement of the flow from one point to another more accurately in cases when the distance between points is relatively big, and the flow movement speed is relatively small. Such situation arises when a traffic lane is divided into long segments, and the flow movement speed is close to 0 m/s but is not equal to 0. The longer the distance between points and the smaller the flow movement speed on the segment are, the longer the delay time is.

Equation (3) member $\left(\frac{v_{i-1}\left(t - \tau_{i-1,i}\right)}{L_{i-1,i}} \right) \cdot v_i(t)$ describes traffic flow acceleration at point '*i*–1' and its impact on the flow speed at point '*i*'; meanwhile member $\left(\frac{1}{2} \frac{v_i(t) + v_{i+1}(t)}{L_{i+1,i}} \right) \cdot v_i(t)$ describes average traffic flow acceleration between points '*i*' and '*i*+1'. The latter member also takes into account the flow condition at the point subsequent to point '*i*' in the direction of the flow movement. If traffic flow speed '*i*+1' decreases, the speed at point '*i*' decreases as well, and vice versus if the speed at point '*i*+1' increases, the speed at point '*i*' increases as well.

Equation (3) member $\left(1 - \frac{k_{i+1}(t)}{k_{\max,i+1}} \right)^{m_1}$ describes traffic flow acceleration change between points '*i*' and '*i*+1'. If vehicle concentration on the preceding traffic lane segment is low, the value of the coefficient approximates to 1; when concentration is high, it approximates to 0. Therefore, when concentration on the preceding road segment increases, vehicle flow speed on road section '*i*' decreases.

Equation (3) member $1 - \frac{k_i(t)}{k_{\max,i}}$ does not allow the system to overfill at point '*i*'. When flow concentration value approximates to k_{max}, the value of the above-mentioned member approximates to 0. Meanwhile, member $\left(\frac{v_i(t)}{v_{\max,i}}\right)\cdot e^{\left(\gamma_3\left(\frac{k_i(t)}{k_{\max,i}}\right)^{m_2}\right)\cdot\left(\frac{v_i(t)}{v_{\max,i}}\right)}$ takes into account the number of vehicles at point '*i*' and directly impacts on the concentration value at point '*i*'.

The number of vehicles on a traffic lane segment is calculated according to the following formula:

$$N_e = \int_{x_i}^{x_j} k(x)dx, \quad (6)$$

where: x_i, x_j – boundary points of a traffic lane segment, $k(x)$ – traffic flow concentration on a traffic lane segment.

Change in the number of vehicles on a traffic lane segment is described as follows:

$$N_i(t) = N_i(t) + \int_{t_i}^{\Delta t + t_i} q_i(t)dt, \quad (7)$$

where: $q_i(t)$ – vehicle flow rate on traffic lane segment '*i*'.

In the text below an example of applying the mathematical model of lumped parameters of traffic flows is presented.

3.2. Equations for Description of ICE emissions

Modelling of automobile combustion products is rather complicated. The authors Mansha *et al.* (2010) and Descombes *et al.* (2003) investigate processes occurring in the internal combustion engine chamber in detail. The articles present programming packages used to simulate combustion processes and one can find out that the simulation of combustion processes is complicated and have a lot of random parameters which may influence on the simulation result. The most commonly used engines were generalised and their emissions were measured by autors (KTI – Institite for Trasnport Sciences) of document ÚTMUTATÓ… (2006). The presented article contains these experimental data used to describe engines' emissions, which are presented in Table 1.

According to the statistical data presented by SE 'Regitra' (*http://www.regitra.lt*), the number of motor vehicles registered in Lithuania are presented in Table 2. Lithuanian (Lietuvoje įregistruotų… 2011) and Hungarian (Markovits-Somogyi, Torok 2010) vehicle fleetcare is very similar so for the modelling is used Lithuanian vehicle fleet.

The data presented in Table 2 show that M1 and N1 category vehicles, according to the data of 1 January 2011 (Lietuvoje įregistruotų… 2011), make up 95.11% of registered vehicles; therefore, vehicular ICE emissions measured at different vehicle movement speeds are used for mathematical modelling (Table 1).

The dimensions of the values in Table 1 are measured in g/h and in g/km. The values from Table 1 are converted from g/h to g/s and are used to build the regression equations Figs 3–7. These functions describing ICE emissions, which depend on one variable parameter, i.e. speed. Programming package Matlab was used to write functions. The method of writing each function is different because the curve which complies with the most measured parameters was sought for.

Functions describing ICE emissions, coefficients' values, written functions and graphical expressions of the measured values are presented in Figs 3–7. The compliance of regression functions with the measured data is 98–99% for all presented functions.

These functions are included in the mathematical model of the road traffic flow and may be used when forecasting or investigating road pollution dynamics.

Table 1. Automobile emissions dependences on vehicles' movement speed (ÚTMUTATÓ… 2006)

Speed, km/h	Emission measuring units	Passenger Cars					Heavy Goods Vehicles				
		Emission factors for 2010 – forecast by the Hungarian dataset (2005)									
		CO	CH(FID)	NO_x	PM	CO_2	CO	CH(FID)	NO_x	PM	CO_2
0	g/h	69.5	4.975	2.11	0.357	1554	61.0	7.80	21.35	1.98	6631.5
5	g/km	13.90	0.995	0.422	0.0714	310.8	12.20	1.560	4.27	0.396	1326.3
10	g/km	11.00	0.900	0.416	0.0597	262.7	10.20	0.611	3.84	0.321	1040.0
20	g/km	7.12	0.714	0.394	0.0439	203.0	7.46	0.423	3.13	0.250	808.7
30	g/km	5.33	0.590	0.405	0.0351	171.7	5.86	0.285	2.83	0.221	716.5
40	g/km	3.97	0.435	0.411	0.0292	154.9	4.96	0.209	2.76	0.206	658.3
50	g/km	3.14	0.418	0.427	0.0255	148.0	4.18	0.166	2.73	0.195	635.6
60	g/km	2.37	0.416	0.486	0.0247	147.4	3.70	0.140	2.86	0.194	633.1
70	g/km	1.72	0.392	0.556	0.0249	151.0	3.18	0.125	3.13	0.191	660.2
80	g/km	1.52	0.379	0.623	0.0263	156.5	2.78	0.124	3.55	0.201	719.4
90	g/km	1.76	0.418	0.668	0.0286	165.6	3.17	0.126	4.13	0.227	822.6
100	g/km	2.07	0.433	0.724	0.0316	178.4	3.96	0.131	5.06	0.256	990.4
110	g/km	2.72	0.442	0.782	0.0345	194.3	–	–	–	–	–

Table 2. The number of road vehicles registered in Lithuania (data of 1 January 2011)

M1 passenger cars (units)	N1 heavy goods vehicles up to 3.5 t (units)	M2 A buses up to 5t (units)	N2 heavy goods vehicles over 3.5 t up to 12 t (units)	M3 buses over 5 t (units)	N3 heavy goods vehicles over 12 t (units)	Total (units)
1734047	95930	7690	33628	7054	45651	1924000
M1+N1= 1829977		M2+N2= 41318		M3+N3= 52705		
95.11%		2.15%		2.74%		100%

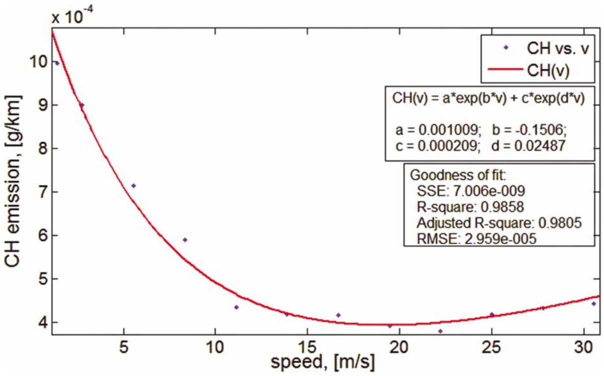

Fig. 3. Dependence of internal combustion engine CH emission on speed

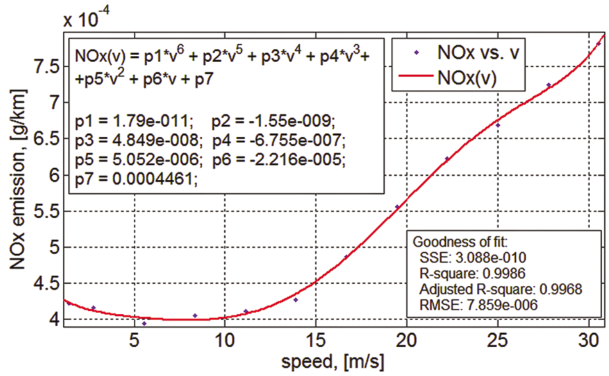

Fig. 6. Dependence of internal combustion engine NO_x emission on speed

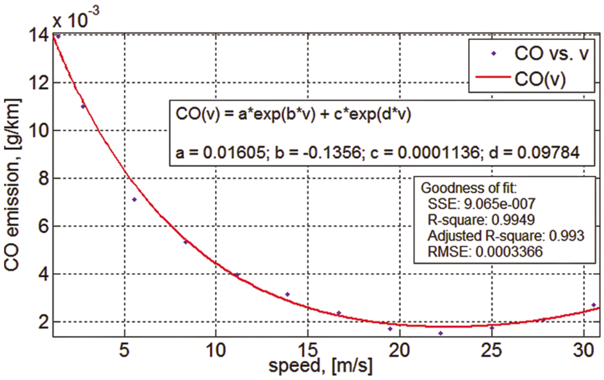

Fig. 4. Dependence of internal combustion engine CO emission on speed

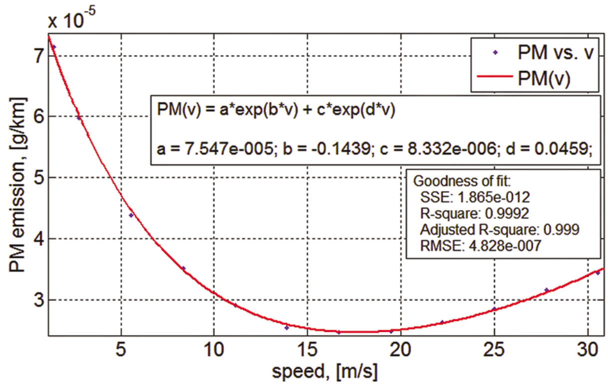

Fig. 7. Dependence of internal combustion engine PM emission on speed

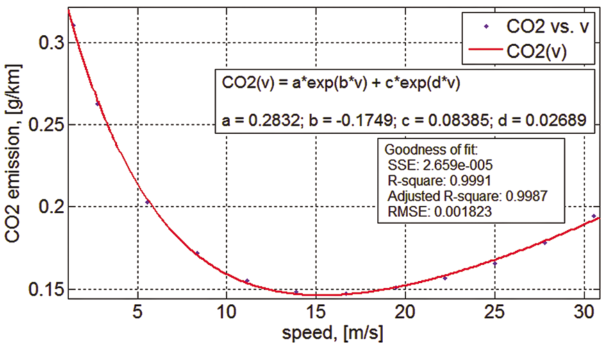

Fig. 5. Dependence of internal combustion engine CO_2 emission on speed

Regression equation and the function of the CO_2 emissions are shown in Fig. 5. Red line is negative when speed is close to 0 m/s. There is some limitation used to avoid inaccuracies in modelling. The speed value could variate from 0.1 m/s to v_{max}. Then the lowest CO_2 emission quantity is not less than 0.06 g/m. The emissions are calculated in two different cases. When the speed is less or equal to 0.1 m/s then emissions values from Table 1 are taken in g/h. These emission values are recalculated from Table 1 data when the speed is equalt to 5 km/h. When the speed is less or equal to 0.1 m/s then emissions values (see Table 1) are taken in g/h. Examples of such modelling are presented in the next chapter.

4. Modelling Examples

The mathematical model is constructed based on the assumptions presented in Chapter 2. A straight 100 m long road section, at the ends of which boundary conditions of the task are introduced, is taken for modelling. A section of the simulated road is divided into 6 segments. Boundary conditions of the task are introduced at points 1 and 7.

Two tasks are solved. According to the conditions of task 1, the road is filled in the beginning of the simulation, and boundary conditions are as follows: v_1 = 5.0 m/s; v_7 = 0.1 m/s; k_1 = 0.18 veh/m; k_7 =0.18 veh/m in the initial moment k = 0.01 veh/m and v =1.0 m/s at all points.

Modelling results are presented in Figs 8–14.

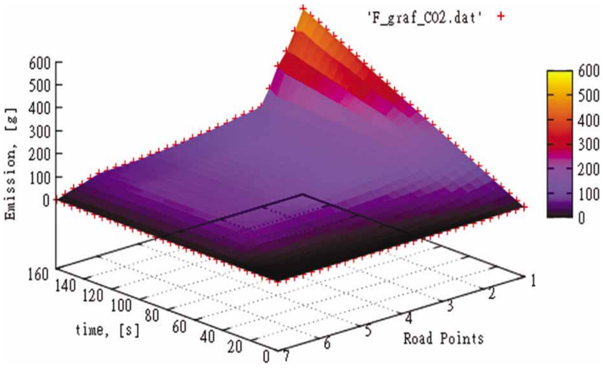

Fig. 8. Dependence of internal combustion engine CO_2 emission on time at road points

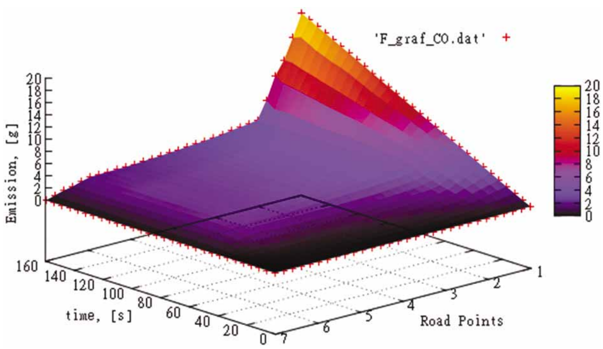

Fig. 9. Dependence of internal combustion engine CO emission on time at road points

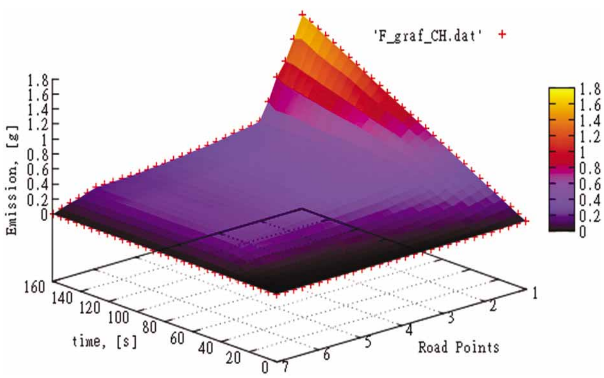

Fig. 10. Dependence of internal combustion engine CH emission on time at road points

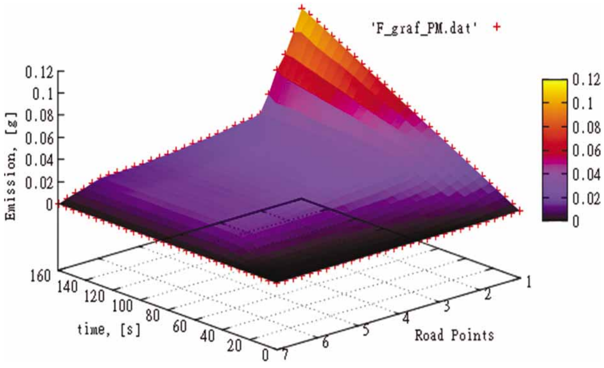

Fig. 11. Dependence of internal combustion engine Pm emission on time at road points

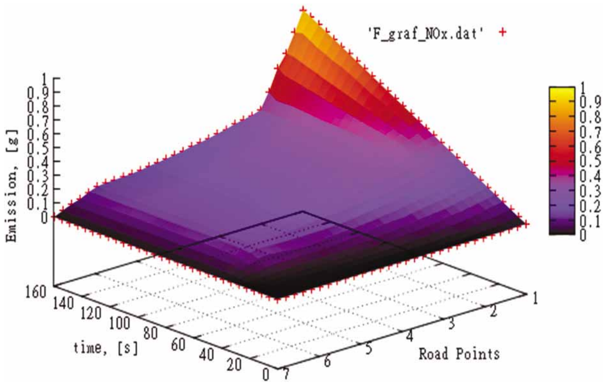

Fig. 12. Dependence of internal combustion engine NO_x emission on time at road points

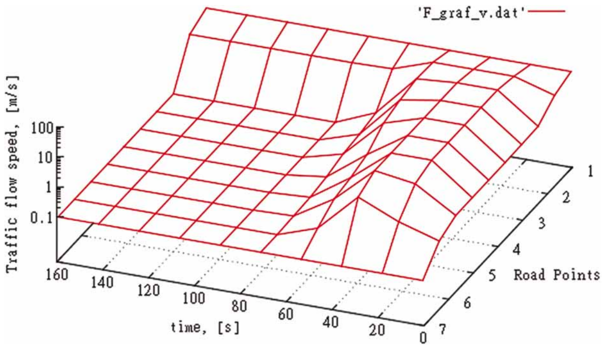

Fig. 13. Dependence of vehicles' flow speed on time at road points

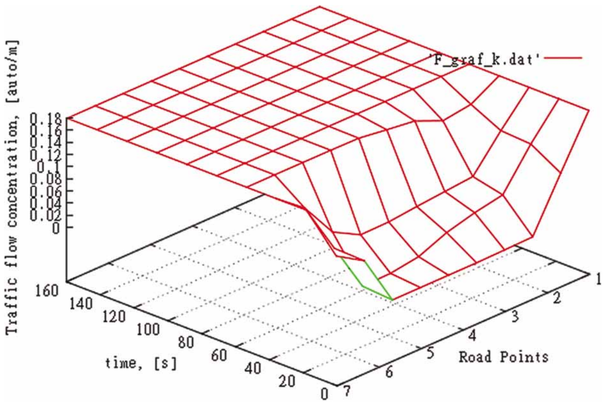

Fig. 14. Dependence of vehicles' flow concentrations on time at road points

According to the conditions of task 2 in the beginning of modelling the road is empty, and boundary conditions are as follows: v_1 = 1.0 m/s; v_7 = 5.0 m/s; k_1 = 0.01 veh/m; k_7 =0.01 veh/m at the initial moment k = 0.18 veh/m; v =1.0 m/s at all road points;

Modelling results are presented in Figs 15–21.

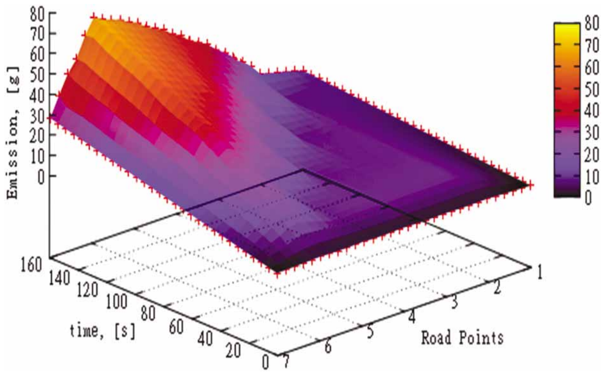

Fig. 15. Dependence of internal combustion engine CO_2 emissions on time at road points

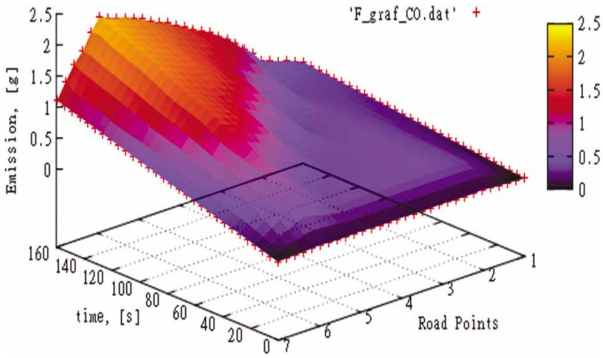

Fig. 16. Dependence of internal combustion engine CO emission on time at road points

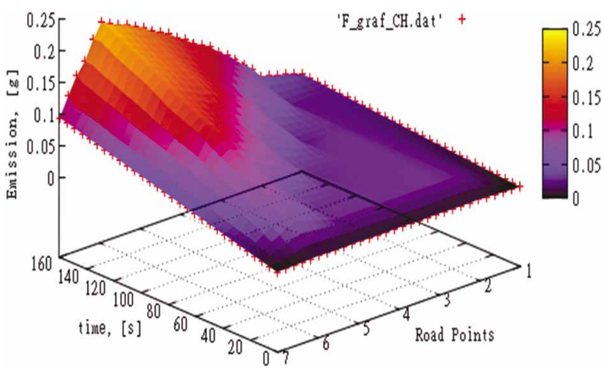

Fig. 17. Dependence of internal combustion engine CH emission on time at road points

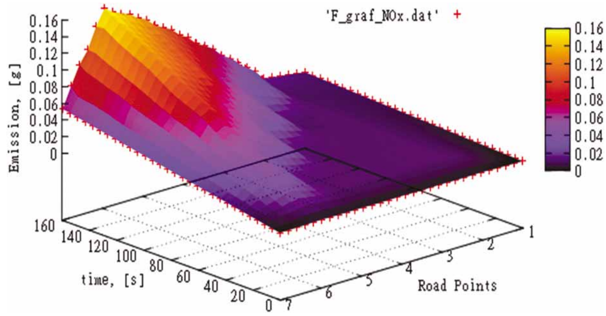

Fig. 18. Dependence of internal combustion engine NO_x emission on time at road points

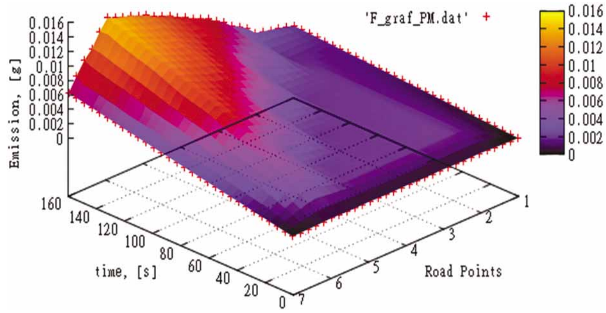

Fig. 19. Dependence of internal combustion engine Pm emission on time at road points

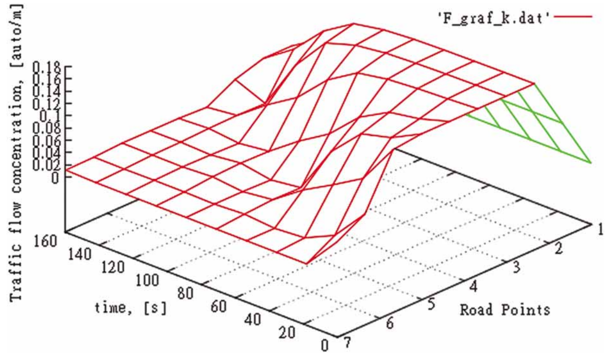

Fig. 20. Dependence of vehicles' flow concentration on time at road points

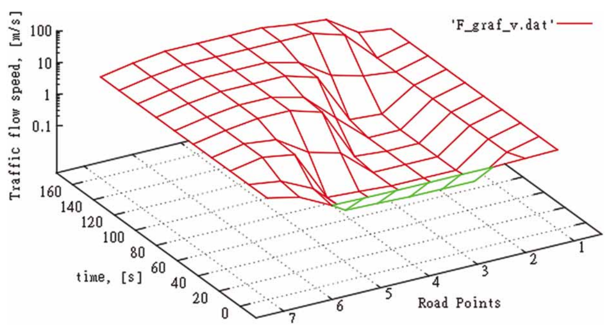

Fig. 21. Dependence of vehicles' flow speed on time at road points

5. Conclusions and Discussion of Simulation Results

Two different cases are simulated.

In the first case, the road segment is empty and during simulation is filled with vehicles.

In the second case, in the beginning of simulation the road segment is filled and becomes empty during simulation. The change of traffic flow parameters k and v at all road points is shown in detail in Figs 13–14 and Figs 20–21.

In the first case, when the road is empty, the speed at road points increases up to the maximum speed limit, and when the road is filled, it decreases to 0 m/s Figs 13–14.

In the second case, concentration at the road points gradually starts to decrease, and the speed gradually increases up to the maximum speed limit from the last road point, Figs 20–21.

ICE emissions at all road points increase because the road segment has some vehicles. In the first case, when in the beginning of modelling the number of vehicles were almost equal to 0, ICE emissions increase during the whole modelling time. In the beginning of modelling this increase is non-linear, and when the road segment is filled, it is linear as speed does not impact on the modelling results, Figs 8–12.

In the second case, when the road segment in the beginning of modelling is filled, ICE emissions increase as well. At the end points of the road, where vehicles start exiting, emissions start decreasing, Figs 15–19. It can be explained by the fact that at the end of the road the number of vehicles decreases and vehicles' speed increases.

Acknowledgements

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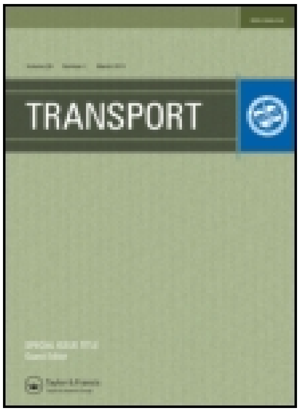

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Mathematical modelling of network traffic flow

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MATHEMATICAL MODELLING OF NETWORK TRAFFIC FLOW

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Abstract. The article describes mathematical models of traffic flows to initiate different traffic flow processes. Separate elements of traffic flow models are made in a way to be connected together to get a single complex model. A model of straight road with different boundary conditions is presented as a separate part of the network traffic flow model. First testing is conducted in case the final point of the whole modelled traffic line is closed and no output from that point is possible. The second test is performed when a constant value of traffic flow speed and traffic flow rate is entered. Mathematical simulation is carried out and the obtained results are listed.

Keywords: traffic flow, intersection, modelling, traffic flow regulation, vehicle.

1. Introduction

Modelling the process of traffic flow was previously studied from different points of view and different mathematical methods were used to describe the same process. It also encounters difficulties in choosing an appropriate method of deriving physical appearance we can notice on our streets and roads. Different authors have different views to the same phenomena and are focusing on different aspects of the same problem (Junevičius and Bogdevičius 2007; Junevičius *et al*. 2007; Berezhnoy *et al*. 2007; Akgüngör 2008a and 2008b; Daunoras *et al*. 2008; Yousefi and Fathy 2008; Gowri and Sivanandan 2008; Jakimavičius and Burinskienė 2007 and 2009; Antov *et al*. 2009, Knowles 2008; Gasser 2003; Helbing and Greiner; Knowles 2008 etc.).

All authors have an agreement on basic traffic flow parameters like, traffic flow density, traffic flow rate or the average speed of traffic flow. Besides, a lot of different investigations into the use of traffic flow models to deal with various problems of engineering are carried out, for example in Sivilevičius and Šukevičius (2007) paper.

A comparison of different continuum models has drawn that a number of scientific works were based on fluid dynamic theory and gas – kinetic traffic flow theory. The kinetic traffic flow theory is used for ‘microscopic’ or ‘macroscopic’ traffic flow models. The kinetic traffic flow theory is used in Flötteröd and Nagel (2007), Gning *et al.* (2008), Li and Xu (2008), Prigogine and Herman (1971) works where various approaches to the similar method are discussed. The equations of these models take different values to derive the same process. The kinetic theory was first used by Prigogine and Herman (1971) and co-workers. They suggested an equation analogous to Boltzmann equation. This theory was later criticized by many authors like Tampère (2004) etc. the papers of whose show the experience of Pavery-Fontna who noticed that Prigogine model had inaccuracies comparing the results of modelling and physical experiments. He suggested vehicle desired velocity towards which its actual velocity tends. Later, many authors mainly focused on a better statistical description of the traffic process.

The ‘macroscopic’ theory of traffic flows also can be developed as the hydrodynamic theory of fluids that was first introduced by Lighthill-Whitham and Richards model (Chalons and Goatin 2008; Kim and Keller 2002; Liu *et al.* 2008; Bonzani 2007; Nikolov 2008). They presented one dimensional model analogous to the fluid stream model. This theory was also criticized by such authors as Tampère (2004) and Daganzo and Nagatani (Liu *et al.* 2008) who proposed the lattice method. Nagatani and Nakanishi model took into account that all vehicles were moving at the same time-independent speed and in the same gap between vehicles. This method was improved later by considering the next-nearest neighbour interaction Liu *et al.* (2008).

Plenty of traffic flow models are based on car–following theories supported by the analogues to Newton’s equation for each individual vehicle interacting in a system of vehicles on the road. Different forms of the equa-

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tion of motion give different versions of car-following models. Stimulus, from which response may occur, may be composed of the speed of a vehicle, difference in the speeds of leading and going after the vehicle, distance–headway etc.

Follow-the-leader and optimal-velocity theories are mostly known car-following theories and have been used by Tampère (2004), Kerner and Klenov (2006). Applying these methods, kinetic and fluid dynamic models could be extended to the critical points when the kinetic and fluid theory gives us inaccuracies comparing with experimental data. For example, the car-following theory could comprise the next-nearest neighbour effect in various lattice models, whereas optimal-velocity models give us an opportunity to model different situations, for example interacting vehicles having different characteristics (car and truck) or vehicles with different desired and optimal speeds. Nevertheless, all these improvements face the problems of properly working models or experience difficulties in achieving an appropriate solution.

Another point causing difficulties is the so called 'vehicular chaos' that is an analogue to 'molecular chaos' used in the kinetic theory of gases. The authors investigated such phenomena in their works (Chalons and Goatin 2008; Safanov *et al.* 2000; Kerner and Klenov 2006). Kerner and Klenov (2006) denotes unstable points on the fundamental diagram. These points indicate minimal density of growing infinite small fluctuations and express a zone for speed variation depending on vehicular density.

A similar zone for speed variation is presented in works by Chalons and Goatin (2008), Safanov *et al.* (2000). The authors derived alternate vehicle transition to the cases of unstable zones. These models clearly explain empirical data on the brake–down points of the fundamental diagram.

2. Description of Traffic Flow Mathematical Model

To model traffic flow, an equation system taking into account two parameters is used: traffic flow speed and traffic flow density. These parameters are calculated on each point of the road and information on the previous and next point of some road mesh is considered (Fig. 1).

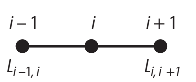

Fig. 1. A **s**cheme for deriving traffic flow values at each traffic line point

At each point '*i*' equations 1 and 2 are derived. Equation 1 derives variations in traffic flow speed and equation 2 derives variations in concentration at each point *i*.

$$\dot{v}_i = p_{in,i}(t)\cdot r_{v_i,in.,i}\cdot\left(\frac{v_{i-1}(t-\tau_{i-1,i})}{L_{i-1,i}}\right)\cdot\left(1-\frac{k_i(t)}{k_{\max,i}}\right)\cdot v_i(t)+$$

$$f_i(k_{i+1,i})-p_{out,i}(t)\cdot r_{v_i,out}\cdot\left(\frac{1}{2}\frac{v_i(t)+v_{i+1}(t)}{L_{i+1,i}}\right)\cdot$$

$$\left(1-\frac{k_{i+1}(t)}{k_{\max,i+1}}\right)^{m_1}\cdot v_i(t)-\left(\frac{v_i(t)}{v_{\max,i}}\right)\cdot e^{\left(\gamma_3\left(\frac{k_i(t)}{k_{\max,i}}\right)^{m_2}\right)\cdot\left(\frac{v_i(t)}{v_{\max,i}}\right)}; \qquad (1)$$

$$\dot{k}_i = p_{in,i}(t)\cdot r_{k_i,in.,i}\cdot\left(1-\frac{k_i(t)}{k_{\max,i}}\right)\cdot\left(\frac{q_{i-1}(t-\tau_{i-1})}{q_{\max,i-1}}\right)\cdot k_i(t)-$$

$$p_{out,i}(t)\cdot r_{k_i,out,i}\cdot\left(1-\frac{k_{i+1}(t)}{k_{\max,i+1}}\right)\cdot\left(\frac{q_i(t)}{q_{\max,i}}\right)\cdot k_i(t), \qquad (2)$$

where: $r_{v_i,in}$, $r_{k_i,in}$, $r_{k_i,out}$ and $r_{v_i,out}$ – parameters are taken from empirical data; $v_{\max}$ – the maximal possible value of traffic flow speed at each point; L_i – road segment depending on point '*i*'; $k_{\max}$ – the maximal possible value of traffic flow density at point '*i*'; $q_{\max}$ – the maximal possible traffic flow rate at point '*i*', q_i – the calculated traffic flow rate; $q_{in,i}$, $q_{out,i}$ – the probability of flow splitting or connecting at some traffic line intersecting point (It means that traffic flow could split between several traffic lines or be diverted to some exact traffic line or connected to one from several separate traffic lines. Depending on time, this parameter could be a constant or a function. It could be used as a control function to model traffic flow intersections, traffic accidents and other perturbations that could occur on the road network); $f_i(k_{i+1,i})$ – is some function depending on parameter *k*:

$$f_i=\begin{cases} e^{\gamma_2\left(1-\frac{\varepsilon_{i,i+1}}{\varepsilon_{i,i}}\right)\cdot\varepsilon_{i,i}\cdot sign(p_{out,i}(t))\cdot sign\left(\left(1-\frac{\varepsilon_{i,i+1}}{\varepsilon_{i,i}}\right)\right)}, & \varepsilon_{i,i}>\varepsilon_{i,i+1}, and\ \varepsilon_{i,i}>0 \\ 0, \text{ otherwise.} & \end{cases} \qquad (3)$$

This function takes into account the state of the road segment in front of point '*i*'.

Other coefficients are $\gamma_3=5.5$, $\gamma_2=2.5$, $m_1=6$, $m_2=10$.

$$\varepsilon=\frac{k_i}{k_{\max}}. \qquad (4)$$

Some explanations about the members of equations (1) and (2) are given below.

These are the members of equation 1:

- Member $\left(\frac{v_{i-1}(t-\tau_{i-1,i})}{L_{i-1,i}}\right)\cdot v_i(t)$ describes traffic flow acceleration at point '*i* – 1' and member $\left(\frac{1}{2}\frac{v_i(t)+v_{i+1}(t)}{L_{i+1,i}}\right)\cdot v_i(t)$ specifies the average traffic flow acceleration.
- Member $\left(1-\frac{k_{i+1}(t)}{k_{\max,i+1}}\right)^{m_1}$ shows variations in the acceleration of traffic flow between points '*i*' and '*i* + 1'.
- Member $1-\frac{k_i(t)}{k_{\max,i}}$ characterizes traffic flow fulfilling point '*i*'.

- Member $\left(\frac{v_i(t)}{v_{\max,i}}\right)\cdot e^{\left[\gamma_3\left(\frac{k_i(t)}{k_{\max,i}}\right)^{m_2}\right]\cdot\left(\frac{v_i(t)}{v_{\max,i}}\right)}$ takes into account the amount of vehicles at point '*i*' and is subject to concentration value at point '*i*'.

These are the members of equation 2:

- Member $1-\frac{k_i(t)}{k_{\max,i}}$ considers traffic concentration at point '*i*' and member $1-\frac{k_{i+1}(t)}{k_{\max,i+1}}$ considers traffic flow concentration at point '*i* + 1'. It means that in case a road in front of point '*i*' is occupied, there is no possibility of entering the road segment in front of point '*i*'.
- Member $\frac{q_{i-1}(t-\tau_{i-1})}{q_{\max,i-1}}$ takes into account traffic flow rate at point '*i* – 1' which means that at point '*i* – 1', there should be some quantity of vehicles that can enter point '*i*'; otherwise the value of traffic flow rate becomes equal to 0. The delay of traffic flow rate that comes from point '*i* – 1' to point '*i*' is also regarded.
- Member $\frac{q_i(t)}{q_{\max,i}}$ shows outgoing traffic flow rate from point '*i*' to point '*i* + 1'.

The quantity of vehicles at each road segment could be calculated by the equation:

$$N_e = \int_{x_i}^{x_j} k(x)\,dx, \qquad (5)$$

where: $x_{i,j}$ – traffic line segment boundary points; $k_{i,j}$ – concentration values at boundary points.

Variance in the quantity of vehicles at each road segment could be derived by the equation:

$$N_i(t) = N_i(t) + \int_{t_i}^{\Delta t + t_i} q_i(t)\,dt\,. \qquad (6)$$

3. Model Description. Numerical Experimental Study

Two cases of mathematical experiment are presented.

Case 1.

To model traffic flows in this paper, the following considerations are required. First, it is acknowledged that the part of the road between two intersections is divided into some intervals e_i (Fig. 2).

Each element has two points at the ends of the interval. Two elements are connected at the same point, so each element has two points that belong to two different elements.

An exception is the first and the last point of the road part that is under investigation as these points are road input and road output respectively.

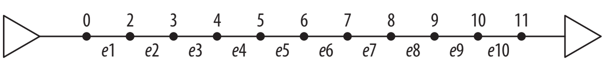

Fig. 2. The structure of creating a part of one way road

The number of points is 11 (10 elements); the length of the road is L = 1 km (Fig. 2).

Boundary conditions at the first and final points are:

- Traffic flow rate:
  $q(x=0,t)=q_1=0.5$ veh/s;
  $q(x=L,t)=q_{11}=0$ veh/s;
- Traffic velocity:
  $v(x=0,t)=v_1=13.888$ m/s $=50$ km/h;
  $v(x=L,t)=v_{11}=0$ m/s.
- Initial conditions:
  $v_i(t=0)=10^{-4}$ m/s; $k_i(t=0)=10^{-4}$ veh/m;
  $i=2,\dots,10$.

Velocity, traffic flow rate and flow density rate are shown in Fig. 3, 4 and 5.

The dependency of a total number of vehicles on the road on time is shown in Fig. 6.

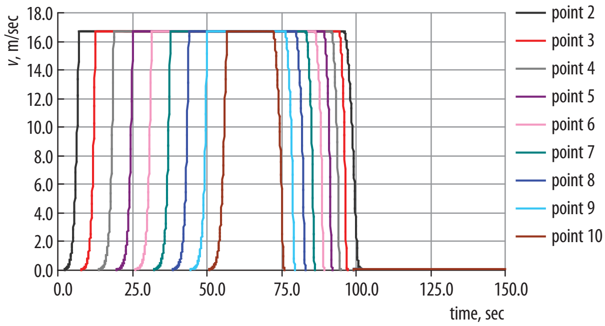

Fig. 3. The dependency of flow velocity on time at each point '*i*'

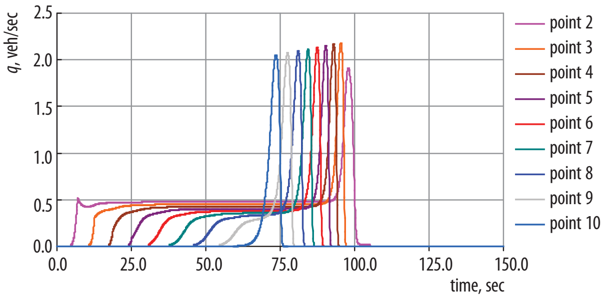

Fig. 4. The dependency of the traffic flow rate on time at each point '*i*'

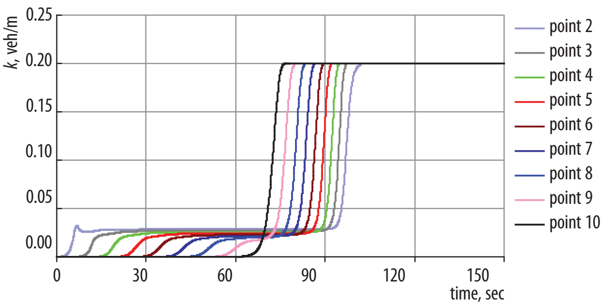

Fig. 5. The dependency of traffic concentration on time at each point '*i*'

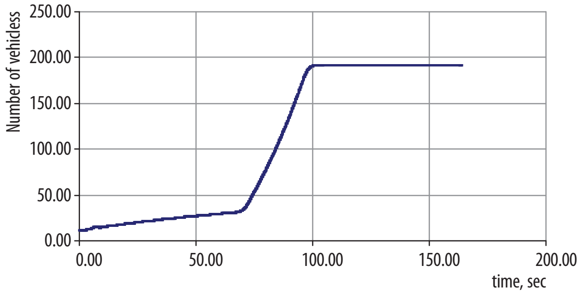

Fig. 6. The dependency of a total number of vehicles on the road on time

The end of the road is closed so the vehicles enter the road but do not leave it. Estimating the result of simulation shows that the road should be overfilled. The investigated part of the road was empty at the start, so speed at the beginning should grow. At a later stage, speed should reach a maximum value. When the road is overfilled, speed should decline to 0.

The data of the conducted mathematical experiment point to the expected results. The empty traffic line was filed with vehicles and the maximum 0.2 veh/m concentration was reached. First, the end of the traffic line was filed up, and then the entire road was filed. Traffic flow speed reaches the maximum value at the beginning of simulation and when concentration starts growing, the speed value reduces to zero.

Traffic flow rate reaches the maximum value and starts declining from the end point of the road. The maximum value of vehicles on the road at peak moment is almost 200 which is the maximum value that could appear on the road when vehicles are bumper to bumper.

Case 2.

Boundary conditions at the first and final points are (Fig. 2):

- Traffic flow rate:
  $q(x=0,t)=q_1=0.5$ veh/s;
  $q(x=L,t)=q_{11}=0.575$ veh/s.
- Traffic velocity:
  $v(x=0,t)=v_1=13.888\ m/s=50$ km/h;
  $v(x=L,t)=v_{11}=10$ m/s.
- Initial conditions: $v_i(t=0)=10^{-4}$ m/s;
  $k_i(t=0)=10^{-4}$ veh/m; $i=2,\ldots,10$.

Velocity, traffic flow rate and flow density rate are shown in Fig. 7, 8 and 9. The dependency of a total number of vehicles on the road on time is shown in Fig. 10.

This test has come up with similar results. This time, the end of the road is open, so all vehicles entering the traffic line could leave it. Speed at the first point is lower than incoming speed in the first case. Traffic flow rate at the end point is higher than that in the first case. Thus, in general, traffic flow rate and concentration decline at each point of the road coming from the first point to the last one and this is due to difference in traffic flow rate under the boundary conditions of the traffic line. Also at the end of simulation, the constant value of vehicles on the road is received. The quantity of vehicles on the road becomes constant at the end of simulation (Fig. 10).

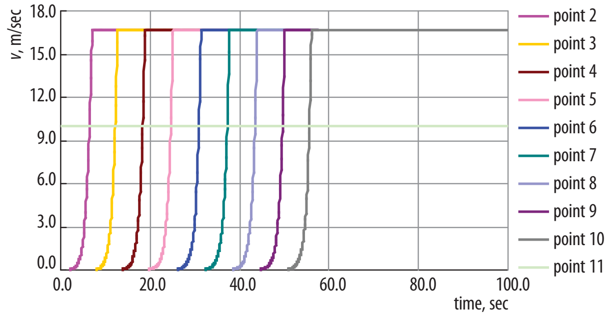

Fig. 7. The dependency of flow velocity on time at each point '*i*'

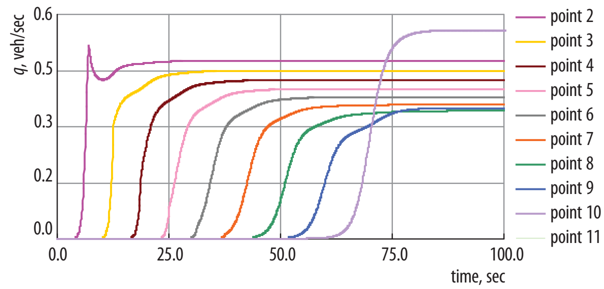

Fig. 8. The dependency of the traffic flow rate on time at each point '*i*'

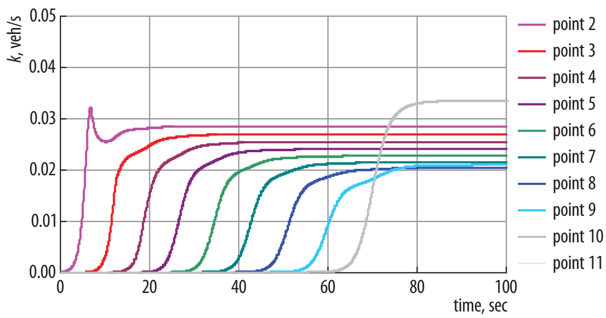

Fig. 9. The dependency of traffic concentration on time at each point '*i*'

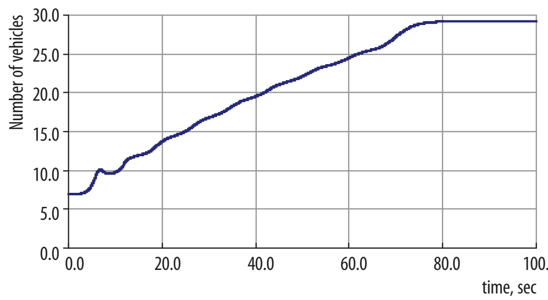

Fig. 10. The dependency of a total number of vehicles on the road on time

4. Conclusions

1. The presented traffic flow model gives theoretically expected results. In each case of simulation, the results are related to boundary conditions. In the first case, the end of the road is closed, $q(x = L, t) = q_{11} = 0$ veh/s, so the number of vehicles on the road increases and reaches the max possible quantity of almost 200 vehicles. In the second case, the end of the road is opened, $q(x = L, t) = q_{11} = 0.575$ veh/s, so after some time, the maximum quantity of almost 30 vehicles is reached.
2. At the beginning of the simulation process, the road was empty. After some time, all segments on the road were filled. First, some concentration and flow values were received. For a while, those values were almost constant. Overfilling the last point starts at a time step of 60 sec. Then, all cells were filed in equal time steps (see Fig. 4 and Fig. 5). Fig. 5 shows that concentration reaches a maximum possible value because the road is closed. Fig. 9 indicates that concentration values are different at all points due to boundary conditions.
3. Traffic flow speed is maximal at all points when concentration is low and begins to increase when concentration starts growing
4. The process of road filling starts from the end point in Case 1 which means that the last road segment was filled first. At a later stage, road segment before him was filed. Thus, the process of filling the entire road starts from the last segment and reaches the first one. Fig. 3, Fig. 4 and Fig. 5 clearly indicate that traffic flow rate and traffic flow concentration change in the same order.

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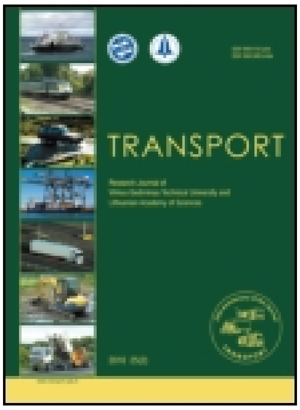

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Determination of traffic flow parameters in different traffic flow interaction cases

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DETERMINATION OF TRAFFIC FLOW PARAMETERS IN DIFFERENT TRAFFIC FLOW INTERACTION CASES

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Abstract. Modelling of straight road section consisting of one traffic line gives the opportunity to simulate "follow the car" system. In general it looks like a line of vehicles, going one after another. Kinetic theory, used in this paper describes traffic flow system as a straight unbroken line with limited flow speed and concentration. Such model also gives the opportunity to derive traffic lines intersections. For example, intersection could be derived like a point with traffic lines coming and outgoing from this point by only changing boundary conditions. Mathematical model is built using characteristic method.

Keywords: Traffic flow, kinetic theory, modelling, characteristic method.

1. Introduction

Traffic lines intersection gives difficulties to model two–way crossroads. To solve this problem at first step we are modelling one-way one line traffic flow intersection with two incoming one outgoing and with one incoming and two outgoing traffic flows. Such model gives solution to classic bottleneck situation on a road. The second step is to determine two line traffic flow model to solve various crossroad problems.

Similar situations were studied in literature [1–6]. Authors in the works [1, 2] based their solutions on kinetic theory, made various traffic flow models on straight road, driver behaviour model. Authors of literature [3] did research into traffic modelling and control using neural networks, authors [6, 7] concentrated on traffic control and regulations in cities.

In this paper at first we derive equations of traffic flow on straight road using characteristic method, and using these results determine traffic flow intersections.

2. Traffic flow model using characteristic method

Basic equations of traffic flow model using kinetic theory are [3]:

$$\frac{\partial k}{\partial t}+V\frac{\partial k}{\partial x}+k\frac{\partial V}{\partial x}=0, \tag{1}$$

$$k\frac{\partial V}{\partial t}+kV\frac{\partial V}{\partial x}+\Theta\frac{\partial k}{\partial x}=k\left\langle\frac{dv}{dt}\right\rangle_v-k\frac{\partial\Theta}{\partial x}, \tag{2}$$

where: k – concentration; V – traffic flow speed; Θ – speed variation; x – point on a road; v – vehicle speed.

Using characteristic method the road section is divided into pieces of length Δx (Fig 1) [5]. Each piece has information which is concentrated in boundaries. Each peace boundary point takes information: velocity v and concentration k.

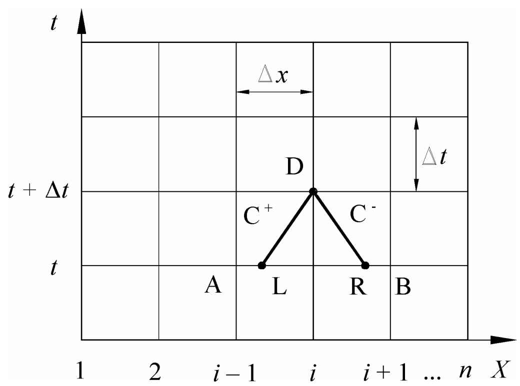

Fig 1. Characteristic net

System of equations (1) and (2) can be written as the system of second-order quasi-linear differential equations:

$$[A]\left\{\frac{\partial u}{\partial t}\right\}+[B]\left\{\frac{\partial u}{\partial x}\right\}=\{f\}, \tag{3}$$

where: $[A]$, $[B]$ are matrices,

$$[A]=\begin{bmatrix}1 & 0\\ 0 & k\end{bmatrix};\ [B]=\begin{bmatrix}V & k\\ \Theta & kV\end{bmatrix};$$

and $\{f\}$ is vector,

$$\{f\}=\left\{k\left\langle\frac{dv}{dt}\right\rangle_v^0-k\frac{\partial\Theta}{\partial x}\right\},$$

which depends on t, x and elements u_i of vector $\{u\}^T = [k, V]$.

Equating the determinant of matrix $[B]-[A]\frac{dx}{dt}$ to zero,

$$\left|[B]-[A]\frac{dx}{dt}\right|=0$$

we shall receive the equation which allows determining $\frac{dx}{dt}$ derivative and determines characteristic direction.

This equation has two various real roots $dx/dt = \lambda_i$ $(i = 1,2)$:

$$C^+:\ \frac{dx}{dt}=V+\sqrt{\Theta}\ ; \tag{4}$$

$$C^-:\ \frac{dx}{dt}=V-\sqrt{\Theta}\ , \tag{5}$$

From equations (1) and (2), using steps listed above, after several transformations receive plus and minus characteristics which have such forms:

$$C^+:\ \frac{dV}{dt}+\frac{\sqrt{\Theta}}{k}\frac{dk}{dt}-g_1=0; \tag{6}$$

$$C^-:\ \frac{dV}{dt}-\frac{\sqrt{\Theta}}{k}\frac{dk}{dt}-g_2=0, \tag{7}$$

where g_i $(i = 1,2)$ are traffic flow speed variations.

$$g_1=\left\langle\frac{dv}{dt}\right\rangle_v-\frac{\partial\Theta}{\partial x};$$

$$g_2=\left\langle\frac{dv}{dt}\right\rangle_v-\frac{\partial\Theta}{\partial x}.$$

The system of two nonlinear algebraic equations is obtained from conditions of compatibility with characteristics:

$$\Phi_1=V_D-V_L+\frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_L+\left(\frac{\sqrt{\Theta}}{k}\right)_D\right]\times$$
$$(k_D-k_L)-\frac{\Delta t}{2}(g_{1L}+g_{1D})=0; \tag{8}$$

$$\Phi_2=V_D-V_R-\frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_R+\left(\frac{\sqrt{\Theta}}{k}\right)_D\right]\times$$
$$(k_D-k_R)-\frac{\Delta t}{2}(g_{2D}+g_{2R})=0; \tag{9}$$

From these equations receive two parameters v and k in each road section boundary point. To determine traffic flow value in road section at first it is needful to determine average flow speed. Generally it could be average vehicle speed.

Equation to calculate traffic flow q has the following form [1]:

$$q=kV\ . \tag{10}$$

Equations listed above could be used for solving different traffic flow problems.

3. Traffic flow intersections

In this chapter three typical intersections schemes will be determined. The first one is a situation when two different traffic flows come into one, as shown in Fig 2.

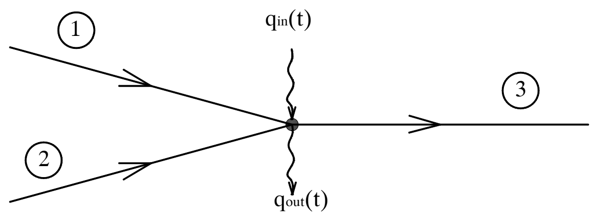

Fig 2. Traffic flow intersection diagram

Intersection is determined as a point at which three separate flows cross. In this case to model interflow two C^+ and one C^- characteristics and additionally one input and one output leaks are used. Equations of this model could be written in the following form:

$$C^+:\ \Phi_1=V_{D,1}-V_{L,1}+\frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_{L,1}+\left(\frac{\sqrt{\Theta}}{k}\right)_{D,1}\right]\times$$
$$(k_{D,1}-k_{L,1})-\frac{\Delta t}{2}(g_{1L,1}+g_{1D,1})=0; \tag{11}$$

$$C^+:\ \Phi_2=V_{D,2}-V_{L,2}+\frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_{L,2}+\left(\frac{\sqrt{\Theta}}{k}\right)_{D2}\right]\times$$
$$(k_{D,2}-k_{L,2})-\frac{\Delta t}{2}(g_{1L,2}+g_{1D,2})=0; \tag{12}$$

$$C^-:\ \Phi_3=V_{D,3}-V_{R,3}-\frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_{R,3}+\left(\frac{\sqrt{\Theta}}{k}\right)_{D,3}\right]\times$$
$$(k_{D,3}-k_{R,3})-\frac{\Delta t}{2}(g_{2D,3}+g_{2R,3})=0. \tag{13}$$

Then traffic flow becomes:

$$p_1(kV)_{D,1} - p_2(kV)_{D,2} - p_3(kV)_{D,3} - p_4 q_{0ut}(t) + p_5 q_{in}(t) = 0, \quad (14)$$

$$\sum_{i=3}^{5} p_i = 1, \quad (15)$$

here: p_i – probability that action may be done.

The second one is a situation when single traffic flow splits into two different traffic flows (Fig 3).

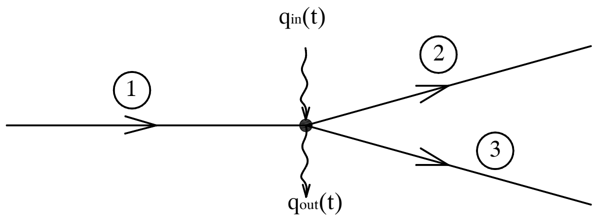

Fig 3. Traffic flow intersection diagram

Intersection also is determined as a point at which three separate flows cross. In this case one C$^+$ and two C$^-$ characteristics and additionally one input and one output leaks are used. Equations of this model could be written in the following form:

$$\text{C}^+\text{:}\ \Phi_1 = V_{D,1} - V_{L,1} + \frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_{L,1} + \left(\frac{\sqrt{\Theta}}{k}\right)_{D,1}\right] \times \left(k_{D,1} - k_{L,1}\right) - \frac{\Delta t}{2}\left(g_{1L,1} + g_{1D,1}\right) = 0; \quad (16)$$

$$\text{C}^-\text{:}\ \Phi_2 = V_{D,2} - V_{R,3} - \frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_{R,2} + \left(\frac{\sqrt{\Theta}}{k}\right)_{D,2}\right] \times \left(k_{D,2} - k_{R,2}\right) - \frac{\Delta t}{2}\left(g_{2D,2} + g_{2R,2}\right) = 0; \quad (17)$$

$$\text{C}^-\text{:}\ \Phi_3 = V_{D,3} - V_{R,3} - \frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_{R,3} + \left(\frac{\sqrt{\Theta}}{k}\right)_{D,3}\right] \times \left(k_{D,3} - k_{R,3}\right) - \frac{\Delta t}{2}\left(g_{2D,3} + g_{2R,3}\right) = 0. \quad (18)$$

Then traffic flow becomes:

$$p_1(kV)_{D,1} - p_2(kV)_{D,2} - p_3(kV)_{D,3} - p_4 q_{0ut}(t) + p_5 q_{in}(t) = 0; \quad (19)$$

$$\sum_{i=2}^{5} p_i = 1. \quad (20)$$

The third one is intersection of four roads. Each road has two, opposite direction of movement, traffic lines (Fig 4).

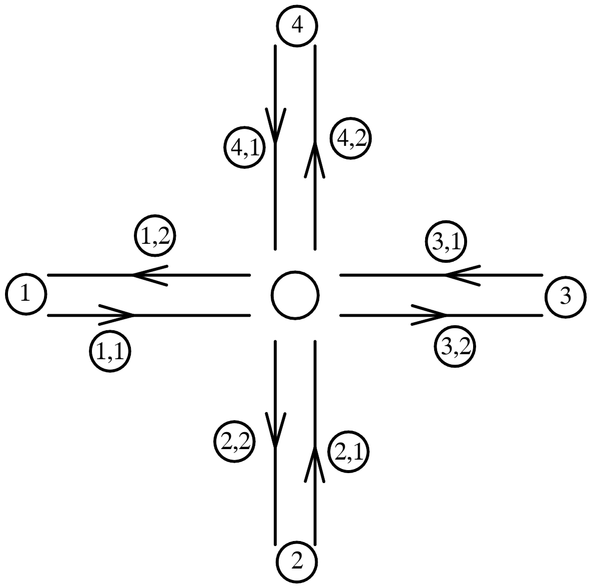

Fig 4. Traffic flow intersection diagram

In this case to determine equations of movement on each road line it is needful to assign C$^+$ characteristic on outgoing traffic line and C$^-$ characteristic on ingoing traffic line. Boundary condition of each road at intersecting point is built using equations (8) and (9). In primitive form these equations can be written in the following way:

$$\begin{cases} K,1: C^+: \Phi_1 = \Phi_1(V_{D,k,1}, k_{D,k,1}) = 0; \\ K,2: C^-: \Phi_2 = \Phi_2(V_{D,k,2}, k_{D,k,2}) = 0. \end{cases} \quad (21)$$

In this equation index 1 is used for incoming and index 2 is used for outgoing traffic flows.

Each of four roads is derived in the same order so in the end receiving equation system from 8 equations.

When boundary conditions are set, the next step is to balance incoming and outgoing flows. Traffic flows balance derivation then takes the following form:

$$\Phi_9 = \sum_{k=1}^{4} p_{k,1}(kV)_{k,1} - \sum_{k=1}^{4} p_{k,2}(kV)_{k,2} = 0. \quad (22)$$

Now there are all 9 equations and it is almost possible to get a solution. The final step to derive equations is to set correct probability laws for all traffic lines.

Probability equation for incoming flow in general form is expressed as:

$$p_{1,1} + p_{1,2,2} + p_{1,3,2} + p_{1,4,2} = 1;$$
$$p_{2,1} + p_{2,2,2} + p_{2,3,2} + p_{2,4,2} = 1;$$
$$p_{3,1} + p_{3,2,2} + p_{3,3,2} + p_{3,4,2} = 1;$$
$$p_{4,1} + p_{4,2,2} + p_{4,3,2} + p_{4,4,2} = 1;$$

$$p_{k,1} + \sum_{i\check{z}1}^{4} p_{k,i,2} = 1; \quad i \neq k. \quad (23)$$

Probability equation for outgoing flow in general form will be as follows:

$$p_{1,2,2} + p_{3,2,2} + p_{4,2,2} = 1;$$
$$p_{1,3,2} + p_{2,3,2} + p_{4,3,2} = 1;$$
$$p_{1,4,2} + p_{2,4,2} + p_{3,4,2} = 1;$$
$$p_{2,1,2} + p_{3,1,2} + p_{4,1,2} = 1;$$

$$\sum_{i=1}^{4} p_{k,i,2} = 1;\ i = 1, .., 4;\ i \neq k. \quad (24)$$

Equations (23) and (24) show that incoming flow from any of four roads can be splited and steered to all three, to two or one outgoing lines. This could be done by changing probability values from 0 to 1. Probability can be taken like the opportunity of choice. Then model may be reorganized so as to minimise or maximise the desired parameter. Or probability may be taken as a strict rule and in that case it is possible to model traffic–light controlled crossroads.

Solution of such equations has quite a simple form. In general it can be as follows:

$$\begin{cases} k_i^{t+\Delta t} = k_i^t + dk; \\ v_i^{t+\Delta t} = v_i^t + dv. \end{cases} \quad (25)$$

where:

$$\begin{Bmatrix} dk \\ dv \end{Bmatrix} = -\begin{bmatrix} \frac{\partial \Phi_1}{\partial k} & \frac{\partial \Phi_1}{\partial v} \\ \frac{\partial \Phi_2}{\partial k} & \frac{\partial \Phi_2}{\partial v} \end{bmatrix}^{-1} \begin{Bmatrix} \Phi_1 \\ \Phi_2 \end{Bmatrix}. \quad (26)$$

These equation systems are used to derive the solution for each section boundary point. To get a solution it is very important to take correct time step of calculation Δt. Time interval Δt is related to road section Δx. To get the right solution it is important to sustain the specification [5]:

$$Cr = \frac{\Delta t}{\Delta x} \leq 1. \quad (27)$$

In the end, such or similar model could be used to solve problems in various road intersection cases, in uncontrolled traffic flows, and to solve problems related to traffic control in cities.

4. Conclusions

1. Using characteristic method and kinetic theory it is possible to make traffic line intersection models in quite a simple way and the models are suitable to use for solving various problems.
2. Intersection models in general are similar to other models, but in this case it is possible to determine the dynamic system of long straight road sections intersecting each other.

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Roundabout Traffic Flow Intersection

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Abstract

Modelling of straight road section consisting of one traffic line gives the opportunity to simulate "follow the car" system. In general it looks like a line of vehicles, going one after another. Kinetic theory, used in this paper describes traffic flow system as a straight unbroken line with limited flow speed and concentration. Such model also gives the opportunity to derive traffic lines intersections. For example, intersection could be derived like point with traffic lines coming and outgoing from this point by only changing boundary conditions. Mathematical model is built using characteristic method.
KEY WORDS: *Traffic flow, kinetic theory, modelling, characteristic method*

1. Introduction

Traffic lines intersection gives difficulties to model two–way crossroads. To solve this problem at first step we are modelling one way one line traffic flow intersection with two incoming one outgoing and with one incoming and two outgoing traffic flows. Such model gives solution to classic bottleneck situation on a road. The second step is to determine two line traffic flow model to solve various crossroad problems.

Similar situations were studied in literature [1–6]. Authors in the works [1, 2] based their solutions on kinetic theory, made various traffic flow models on straight road, driver behaviour model. Authors of literature [3] did research in traffic modelling and control using neural networks, authors [6, 7] concentrated on traffic control and regulations in cities.

In this paper at first we derive equations of traffic flow on straight road using characteristic method, and using these results determine traffic flow intersections.

2. Traffic flow model using characteristic method

Basic equations of traffic flow model using kinetic theory are [3]:

$$\frac{\partial k}{\partial t}+V\frac{\partial k}{\partial x}+k\frac{\partial V}{\partial x}=0, \qquad (1)$$

$$k\frac{\partial V}{\partial t}+kV\frac{\partial V}{\partial x}+\Theta\frac{\partial k}{\partial x}=k\left\langle\frac{dv}{dt}\right\rangle_v-k\frac{\partial\Theta}{\partial x}, \qquad (2)$$

here: k – concentration; V – traffic flow speed; Θ - speed variation; x – point on a road; v – vehicle speed.

Using characteristic method the road section is divided into pieces of length Δx (Fig 1) [5]. Each piece has information which is concentrated in boundaries. Each peace boundary point takes information: velocity v and concentration k.

System of equations (1) and (2) can be written as the system of second-order quasi-linear differential equations:

$$[A]\left\{\frac{\partial u}{\partial t}\right\}+[B]\left\{\frac{\partial u}{\partial x}\right\}=\{f\}, \qquad (3)$$

here: $[A]$, $[B]$ are matrices,

$$[A]=\begin{bmatrix}1 & 0\\ 0 & k\end{bmatrix};\quad [B]=\begin{bmatrix}V & k\\ \Theta & kV\end{bmatrix};$$

and $\{f\}$ is vector,

$$\{f\} = \left\{ \begin{matrix} 0 \\ k\left\langle \frac{dv}{dt} \right\rangle_v - k\frac{\partial \Theta}{\partial x} \end{matrix} \right\}.$$

which depends on t, x and elements u_i of vector $\{u\}^T = [k, V]$.

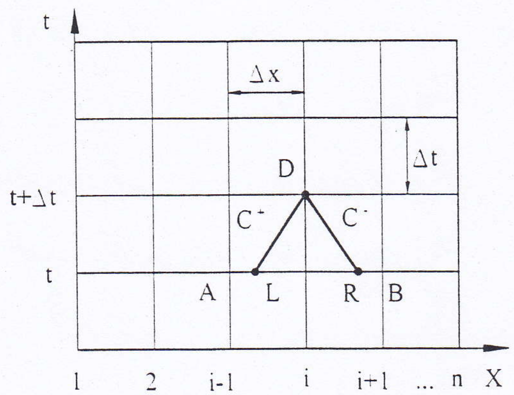


Fig. 1. Characteristic net

Equating the determinant of matrix $[B]-[A]\frac{dx}{dt}$ to zero,

$$\left|[B]-[A]\frac{dx}{dt}\right| = 0.$$

we shall receive the equation which allows determining $\frac{dx}{dt}$ derivative and determines characteristic direction. This equation has two various real roots $dx/dt = \lambda_i$ $(i = 1,2)$:

$$C^+ : \frac{dx}{dt} = V + \sqrt{\Theta}; \tag{4}$$

$$C^- : \frac{dx}{dt} = V - \sqrt{\Theta}, \tag{5}$$

From equations (1) and (2), using steps listed above, after several transformations receive plus and minus characteristics which have such forms:

$$\text{C+:} \quad \frac{dV}{dt} + \frac{\sqrt{\Theta}}{k}\frac{dk}{dt} - g_1 = 0; \tag{6}$$

$$\text{C-:} \quad \frac{dV}{dt} - \frac{\sqrt{\Theta}}{k}\frac{dk}{dt} - g_2 = 0. \tag{7}$$

Here g_i $(i = 1,2)$ are traffic flow speed variations.

$$g_1 = \left\langle \frac{dv}{dt} \right\rangle_v - \frac{\partial \Theta}{\partial x};$$

$$g_2 = \left\langle \frac{dv}{dt} \right\rangle_v - \frac{\partial \Theta}{\partial x}.$$

The system of two nonlinear algebraic equations is obtained from conditions of compatibility on characteristics:

$$\Phi_1 = V_D - V_L + \frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_L + \left(\frac{\sqrt{\Theta}}{k}\right)_D\right](k_D - k_L) - \frac{\Delta t}{2}(g_{1L} + g_{1D}) = 0; \tag{8}$$

$$\Phi_2 = V_D - V_R - \frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k}\right)_R + \left(\frac{\sqrt{\Theta}}{k}\right)_D\right](k_D - k_R) - \frac{\Delta t}{2}(g_{2D} + g_{2R}) = 0; \tag{9}$$

From these equations receive two parameters v and k in each road section boundary point. To determine traffic flow value in road section at first it is needful to determine average flow speed. Generally it could be average vehicle speed.

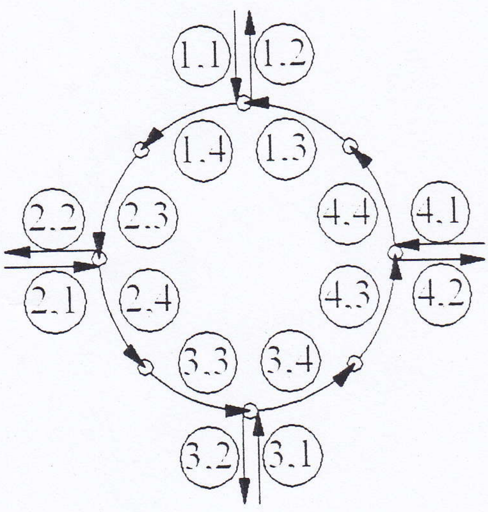
Equation to calculate traffic flow q has the following form [1]:

$$q = kV. \tag{10}$$

Equations listed above could be used for solving different traffic flow problems.

3. Traffic flow intersections

There is chosen intersection with four incoming and four outgoing traffic lines in this chapter (Fig 2a.). Roundabout is divided in to eight parts. Four of them in opposite points are connecting to strait road sections with incoming and outgoing traffic lines. So in the end in each of these four points are two incoming and two outgoing traffic lines and to determine such intersection using characteristic method each incoming line is derived using C^+ and each outgoing line is derived using C^- characteristics. Roundabout road part is derived like straight road part with the same boundary conditions in the ends (Fig. 2b.).



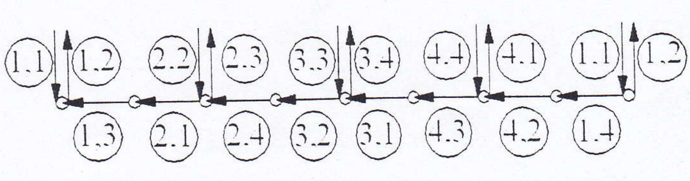


Fig. 2. a) Ring type intersection. b) Simplified ring type intersection.

In the end there is a system of four road intersections of two incoming and two outgoing traffic lines connected to each other and deriving the ring. Additional eight traffic lines hide inflows and outflows to ring type road section. So in that configuration is valid general characteristic method law, each section starts from C^+ characteristic from one side and C^- characteristic from another.

Roundabout traffic flow has priority on incoming straight road sections, also to leave the ring section. In case the flow concentration of ring section point one step ahead and one step behind is law the flow can in leak from outside traffic line. To prevent ring section overload consider ring section top concentration k_{ring} which is equal to jam concentration. Ring type road segment is derived using equations 8,9,10.

To determine equations of movement on each road line it is needful to assign C^+ characteristic on outgoing traffic line and C^- characteristic on ingoing traffic line. Boundary condition of each road at intersecting point is built using equations 8 and 9. In primitive form these equations can be written in the following way:

$$\begin{cases} K,1: C^+ : \Phi_1 = \Phi_1(V_{D,k,1}, k_{D,k,1}) = 0; \\ K,2: C^- : \Phi_2 = \Phi_2(V_{D,k,2}, k_{D,k,2}) = 0; \\ K,3: C^+ : \Phi_1 = \Phi_1(V_{D,k,3}, k_{D,k,4}) = 0; \\ K,4: C^- : \Phi_2 = \Phi_2(V_{D,k,4}, k_{D,k,4}) = 0. \end{cases} \tag{11}$$

In this equation index 1 is used for incoming and index 2 is used for outgoing traffic flows.
Each of four inflow road is derived in the same order so in the end receiving equation system from 8 equations.

When boundary conditions are set, the next step is to balance incoming and outgoing flows. Traffic flow incoming - outgoing balance derivation then takes the following form:

$$\Phi_9 = \sum_{k=1}^{4} p_{k,1}(kV)_{k,1} - \sum_{k=1}^{4} p_{k,2}(kV)_{k,2} = 0. \tag{12}$$

Here: p - probability of an action to be taken.
Probability equation for incoming flow in general form is expressed as:

$$\begin{aligned} &p_{1,1} + p_{1,2,2} + p_{1,3,2} + p_{1,4,2} + p_{1,1,2} = 1; \\ &p_{2,1} + p_{2,1,2} + p_{2,3,2} + p_{2,4,2} + p_{2,2,2} = 1; \\ &p_{3,1} + p_{3,1,2} + p_{3,2,2} + p_{3,4,2} + p_{3,3,2} = 1; \\ &p_{4,1} + p_{4,1,2} + p_{4,2,2} + p_{4,3,2} + p_{4,4,2} = 1; \end{aligned} \tag{13}$$

Probability equation for outgoing flow in general form will be as follows:

$$\begin{aligned} &p_{1,2,2} + p_{3,2,2} + p_{4,2,2} + p_{2,2,2} = 1; \\ &p_{1,3,2} + p_{2,3,2} + p_{4,3,2} + p_{3,3,2} = 1; \\ &p_{1,4,2} + p_{2,4,2} + p_{3,4,2} + p_{4,4,2} = 1; \\ &p_{2,1,2} + p_{3,1,2} + p_{4,1,2} + p_{1,1,2} = 1; \end{aligned} \tag{14}$$

Equations 13 and 14 show that incoming flow from any of four roads can be spited and steered to all three, to two or one outgoing line. This could be done by changing probability values from 0 to 1. Probability can be taken like the opportunity of choice. Then model may be reorganized so as to minimise or maximise the desired parameter.
Solution of such equations has quite a simple form. In general it can be as follows:

$$\begin{cases} k_i^{t+\Delta t} = k_i^t + dk; \\ v_i^{t+\Delta t} = v_i^t + dv. \end{cases} \tag{15}$$

Here:

$$\begin{Bmatrix} dk \\ dv \end{Bmatrix} = -\begin{bmatrix} \dfrac{\partial \Phi_1}{\partial k} & \dfrac{\partial \Phi_1}{\partial v} \\ \dfrac{\partial \Phi_2}{\partial k} & \dfrac{\partial \Phi_2}{\partial v} \end{bmatrix}^{-1} \begin{Bmatrix} \Phi_1 \\ \Phi_2 \end{Bmatrix}. \tag{16}$$

These equation systems are used to derive the solution for each section boundary point. To get a solution it is very important to take correct time step of calculation Δt. Time interval Δt is related to road section Δx. To get the right solution it is important to sustain the specification [5]:

$$Cr = \frac{\Delta t}{\Delta x} \leq 1; \tag{17}$$

In the end, such or similar model could be used to solve problems in various road intersection cases, in uncontrolled traffic flows, and to solve problems related to traffic control in cities.

4. Conclusions

Using characteristic method and kinetic theory it is possible to make traffic line intersection models in quite a simple way and the models are suitable to use for solving various problems.

Intersection models in general are similar to other models, but in this case it is possible to determine the dynamic system of long straight road sections intersecting each other.

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TRAFFIC FLOW DISCRETE MODEL BASED ON HUTCHINSON EQUATION

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Anotacija
Transporto srautų modeliavimas visada buvo gan komplikuotas ir daug kartų nagrinėta anksčiau skirtingais matematiniais metodais. Šiame straipsnyje pristatytas diskretinis metodas, kuriame transporto srautai yra modeliuoti naudojant Hačinsono lygtį transporto priemonių koncentracijai apskaičiuoti. Transporto srauto greitis ir pats srautas apskaičiuoti naudojant paskaičiuotas srauto koncentracijos vertes.

Annotation
Traffic flow modeling is quite complicated and is a lot of times studied before. Is many different mathematical methods studied before. Here in this paper is one discrete method presented. Traffic flow is modeled using Hutchinson equation to simulate concentration variance. Using simulated concentration traffic flow speed variance and flow variance itself is simulated.
KEYWORDS: modeling, traffic flows, mathematical methods

Introduction

From the early beginning traffic flow modeling has difficulties related on reliability. Various mathematical models and integer methods were used. Most of them were based on Kinetic traffic flow theory, neural networks and fuzzy logic theory or some other.

Here In our mathematical model we are using Hutchinson equation [1, 2, 3] and solving equations system using Euler method. This quite simple way gives us possibility of modeling traffic flows on straight road sections or on street networks.

Hutchinson equation based model

To derive traffic flow using Hutchinson equation at firs is necessary to derive traffic line and boundary conditions. In our model modeled traffic section is divided in to sections. Traffic line segment is taken as a straight one-way traffic line the length of witch is L_i and constant for all road segments. This segment from both ends intersects neighboring segments and on intersection points the boundary conditions are the same. First and the last points of all traffic line have different description. This is because of second. Firs point of the modeled road interval is data input point and the last one is the output point. Depending on input and output information various traffic times depended situations could be modeled. These are the points with parameters that are known all the investigating time and could be constant or time depended. According this information the distribution on road section between these points could be derived.

To derive such model we are using equations system [1, 2, 3]:

$$\begin{cases} \dot{x}_1 = s - r\left(1 - \dfrac{x_2}{x_{\max,1}}\right)x_{\tau,1}, \\ \dot{x}_i = r\left(1 - \dfrac{x_i}{x_{\max,i}}\right)x_{\tau,i-1} - r\left(1 - \dfrac{x_{i+1}}{x_{\max,i}}\right)x_{\tau,i}, i = 2..n-1, \\ \dot{x}_n = r\left(1 - \dfrac{x_n}{x_{\max,n}}\right)x_{\tau,n-1}; \end{cases} \quad (1)$$

here. x_i – traffic concentration on road segment i. $x_{max.i}$ – max possible concentration on road segment i. $x_{\tau.i}$ – concentration on road segment i at time step $t-\tau$, r – constant. s – constant. n – road segments quantity. Parameter $x_{\tau.i}$ in each point is coupled all simulation time and after delay time $\tau = \frac{L_i}{\bar{v}}$ is used in equations. This is done because of condition that parameters in two different neighborhood points can not change immediately. Changes from one point travels to another at time step τ which depends on average traffic flow speed.

To calculate traffic flow velocity on each point equation 2 is used [4.8]:

$$v_i = \lambda \ln\left(\frac{x_{max.i}}{x_i}\right) i = 1..n; \qquad (2)$$

here λ constant depending on flow parameters.

To prevent cell overfilling in jammed situations equations system (1) has such limitation:

$$\begin{cases} x_i \geq x_{max}, \rightarrow x_i = x_{max} \\ else, \rightarrow x_i = x_i; \end{cases} \qquad (3)$$

Separate case when the traffic flow in final point is time depended are derived in equation (4). Here in equation of final point time depended variable x_n is inserted.

$$\begin{cases} \dot{x}_1 = s - r\left(1 - \frac{x_2}{x_{max.1}}\right)x_{\tau.1}, \\ \dot{x}_i = r\left(1 - \frac{x_i}{x_{max.i}}\right)x_{\tau.i-1} - r\left(1 - \frac{x_{i-1}}{x_{max.i}}\right)x_{\tau.i}, i = 2..n-1, \\ \dot{x}_n = r\left(1 - \frac{x_n}{x_{max.n}}\right)x_{\tau.n-1} - x_n; \end{cases} \qquad (4)$$

Traffic flow is calculated using general. well known equation [4. 5. 6. 8. 9]:

$$q_i = v_i x_i \qquad (5)$$

Here v – traffic flow speed. x – traffic flow concentration at road segment.

Simulation parameters are as follows: L=50m. $n = 10$. $x_{max.i} = 0.18$. constants values r and s depends on road traffic conditions and needs improvement related with real traffic flow measurements. To solve the equation system Euler method is used [7].

In figures below simulation results are shown.

As figures shows traffic flow has stable and clear interval and only near jammed section quite big disturbances are received. After some small time interval. about these disturbances comes to stable jammed state. Disturbance time step is not big and depends on road segment length. Dependence on other parameters also takes effect. but to have clear results experimental data is needed.

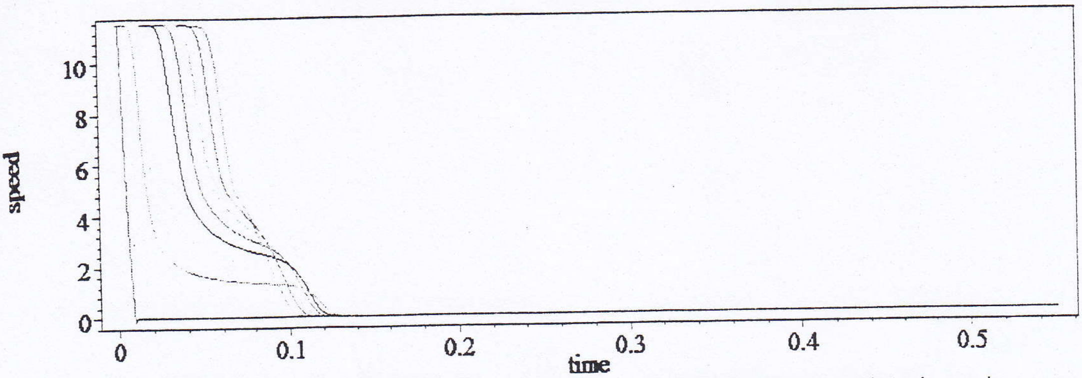

Fig. 1. Traffic flow speed dependence from time when is no output in final point in various points

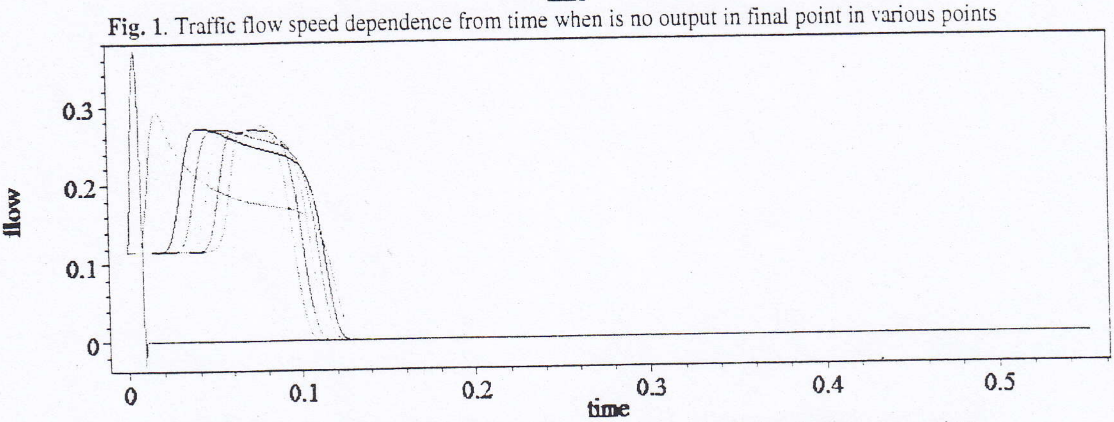

Fig. 2. Traffic flow dependence from time when is no output in final point in various points

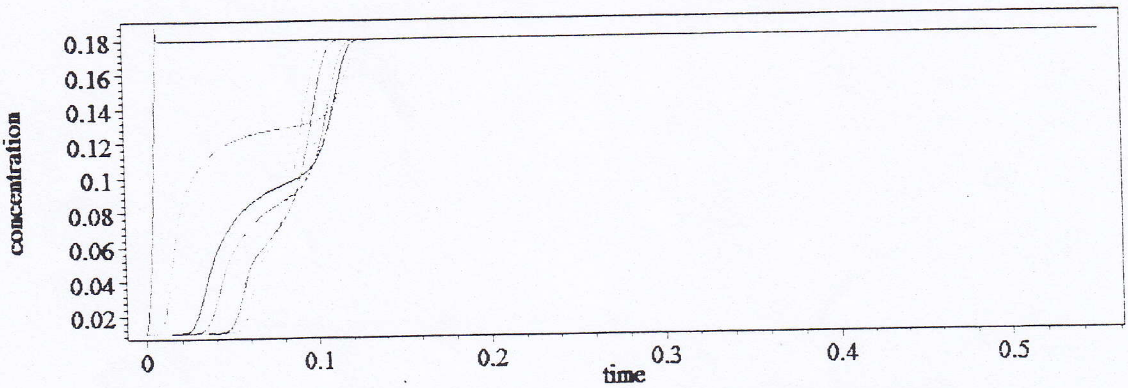

Fig. 3. Traffic flow concentration dependence from time when is no output in final point in various points

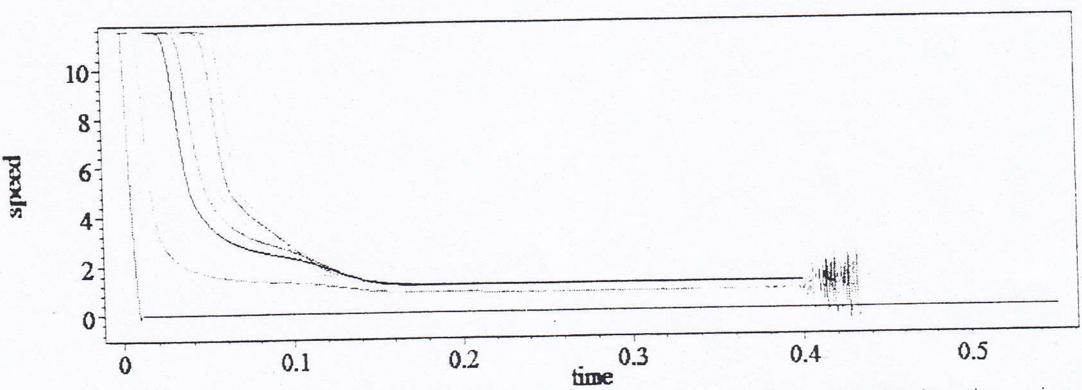

Fig. 4. Traffic flow speed dependence on time when output in final point is depends on time in various points

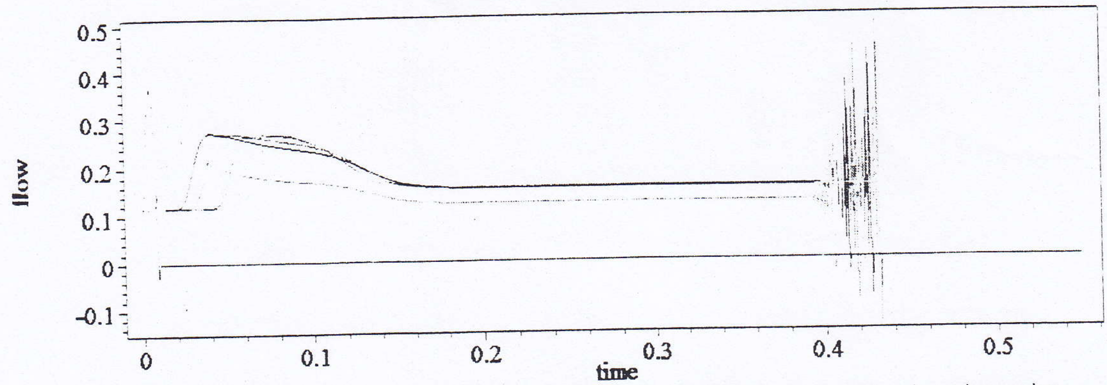

Fig. 5. Traffic flow dependence on time when output in final point is depends on time in various points

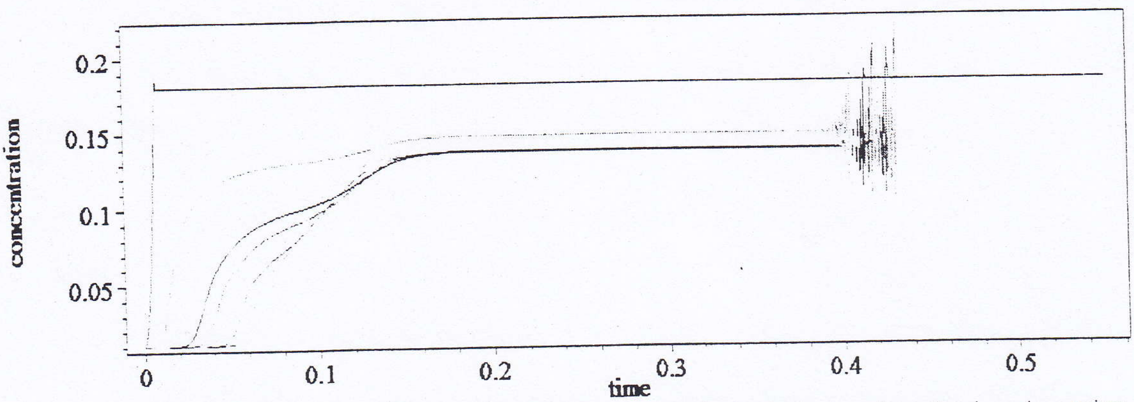

Fig. 6. Traffic flow concentration dependence on time when output in final point is depends on time in various points

Conclusions

1. Simulation results show that such model with a few limitations works quite well. In figures 4. 5. 6 traffic jam situation and on time interval close to jammed flow state unstable equilibrium is seen.
2. Delay time of parameters is computed. so it is possible to model traffic waves. Also model could work when road segments are short so it is usable to model traffic network in a cities.

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S u m m a r y

Traffic flow modeling is quite complicated and is a lot of times studied before. Many different mathematical methods studied before. Here in this paper is one discrete method presented. Traffic flow is modeled using Hutchinson equation to simulate concentration variance. Using simulated concentration traffic flow speed variance and flow variance itself is simulated. Method is suitable to model traffic networks in cities gives unique lock to jammed traffic flow state

A p i b e n d r i n i m a s

Transporto srautų modeliavimas visada buvo gan komplikuotas ir daug kartų nagrinėta anksčiau skirtingais matematiniais metodais. Šiame straipsnyje pristatytas diskretinis metodas. kuriame transporto koncentracija modeliuojama naudojant Hačinsono lygtį transporto priemonių koncentracijai apskaičiuoti. Transporto srauto greitis ir pats srautas apskaičiuoti naudojant paskaičiuotas srauto koncentracijos vertes. Metodas tinkamas modeliuoti transporto srautų tinklą miestuose. suteikia galimybę nagrinėti atvejus kai artėjama prie kamščių susidarymo situacijos.

TRANSPORTO SRAUTŲ MODELIAVIMO TIESIOJE KELIO ATKARPOJE KRITINIAI MOMENTAI

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1. Įvadas

Modeliuojant transporto srautus, dėl ypatingų proceso savybių kyla įvairių problemų juos aprašant matematiškai. Pasirinkus modelio tipą, be kitų parametrų, tokių kaip greitis, kritinė koncentracija, vairuotojo elgesys, kurie pirmiausia pabrėžiami modelius sudarančių autorių darbuose [1, 2, 3, 4, 5, 6], labai svarbu tinkamai nustatyti ir tarpusavyje suderinti pradinius parametrus sprendžiant transporto srautų uždavinius.

Toliau tekste nagrinėjamas kinetine dujų judėjimo teorija paremtas transporto srautų modelis, sprendžiamas charakteristikų metodu. Atkreipiamas dėmesys į probleminius uždavinio sprendimo momentus. Aptariamas pasitelkiamų pataisų tikslingumas ir nurodoma jų teigiama įtaka sprendimo rezultatui.

2. Modeliavimo ypatumai

Sudarytas transporto srauto modelis tiesioje kelio atkarpoje. Kinetinis transporto srautų modelis aprašomas lygtimis [3]:

$$\frac{\partial k}{\partial t} + V\frac{\partial k}{\partial x} + k\frac{\partial V}{x} = 0; \qquad (1)$$

$$k\frac{\partial V}{\partial t} + kV\frac{\partial V}{\partial x} + \Theta\frac{\partial k}{\partial x} = k\left\langle \frac{dv}{dt} \right\rangle_v - k\frac{\partial \Theta}{\partial x}; \qquad (2)$$

čia k – eismo srauto koncentracija; V – transporto srauto judėjimo greitis; Θ – greičio dispersija; x – kelio taško koordinatė; v – automobilio judėjimo greitis.

Atitinkamai pertvarkius lygtis gaunamos pliusinė ir minusinė charakteristikos, kurios naudojamos sudarant sistemos spremdinius:

$$+ \qquad \frac{dV}{dt} + \frac{\sqrt{\Theta}}{k}\frac{dk}{dt} - \left(\left\langle \frac{dV}{dt} \right\rangle_v - \frac{\partial \Theta}{\partial x} \right) = 0; \tag{3}$$

$$- \qquad \frac{dV}{dt} - \frac{\sqrt{\Theta}}{k}\frac{dk}{dt} - \left(\left\langle \frac{dV}{dt} \right\rangle_v - \frac{\partial \Theta}{\partial x} \right) = 0 \,. \tag{4}$$

Integruojant šias lygtis pagal koncentracijos *dk* ir greičio *dv* pokyčius gaunami du lygčių sistemos sprendiniai:

$$\Phi_1 = V_i - V_{i-1} + \frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k} \right)_{i-1} + \left(\frac{\sqrt{\Theta}}{k} \right)_i \right] (k_i - k_{i-1}) - \frac{\Delta t}{2}(g_i + g_{i-1}); \tag{5}$$

$$\Phi_2 = V_i - V_{i+1} - \frac{1}{2}\left[\left(\frac{\sqrt{\Theta}}{k} \right)_{i+1} + \left(\frac{\sqrt{\Theta}}{k} \right)_i \right] (k_i - k_{i+1}) - \frac{\Delta t}{2}(g_i + g_{i+1}); \tag{6}$$

čia $g = \left\langle \frac{dV}{dt} \right\rangle_v - \frac{\partial \Theta}{\partial x}$ – transporto srauto greičio pokytis.

Sprendžiant uždavinį, būtina pasirinkti pakankamai didelę kelio atkarpą ir ja suskaidyti į baigtinius vienodo ilgio kelio segmentus. Kiekvienam segmento taškui formuojamos tokios lygčių sistemos:

$$\begin{cases} v_i^{t+\Delta t} = v_i^t + dv \\ k_i^{t+\Delta t} = k_i^t + dk \end{cases}; \tag{7}$$

$$\text{čia } \begin{Bmatrix} dk \\ dv \end{Bmatrix} = \begin{bmatrix} \frac{\partial \Phi_1}{\partial dk} & \frac{\partial \Phi_1}{\partial dv} \\ \frac{\partial \Phi_2}{\partial dk} & \frac{\partial \Phi_2}{\partial dv} \end{bmatrix}^{-1} \begin{Bmatrix} \Phi_1 \\ \Phi_2 \end{Bmatrix}. \tag{8}$$

Toliau charakteristikų metodu sprendžiant šias lygtis būtina tinkamai aprašyti kraštines sąlygas. Viena iš standartinių uždavinio formų užrašoma taip. Pirmame taške pasirenkamas pastovus greitis ir šio taško reikšmė tokiu laiko momentu skaičiuojama iš minusinės charakteristikos. Paskutiniame nagrinėjamo kelio ruožo taške pasirenkama pastovi, maksimaliai leistina, koncentracija, o greitis šiame taške prilyginamas 0. Galinio taško reikšmės esti tokios, nes pasirenkama prielaida, jog esant ribinei koncentracijai transporto priemonių judėjimas negalimas.

Dar viena sąlyga norint gauti patikimus rezultatus reikalauja tarpusavyje suderinti kelio atkarpos segmento Δx ir laiko žingsnio Δt reikšmes. Čia naudojama Kuranto sąlyga, kuri taikoma sprendžiant uždavinius charakteristikų metodu [2]:

$$\Delta t < \frac{\Delta x}{V}. \tag{9}$$

Išsamiau nagrinėjant lygtis pastebima, jog labai svarbu tinkamai pasirinkti šiuos transporto srautų lygčių parametrus: tai V – transporto srauto greičio vidurkis ir g – transporto srauto greičio pokytis.

Vidutinis srauto greitis, kuris autorių [3, 4, 5] darbuose dažnai sutinkamas kaip transporto srauto greičio vidurkis ar vidutinė greičio reikšmė, turėtų būti imamas kaip didžiausias galimas transporto srauto greitis. Ši prielaida kyla iš tokių sąlygų: sprendžiant charakteristikų metodu būtina pasitelkti pastovų laiko žingsnį Δt ir pastovaus ilgio kelio segmentą Δx. Lygtyse naudojamas parametras $\Theta = V^2 - v^2$, kurio išraiška savo prasme yra tapati dispersijai. Lygtyse naudojamas maksimalus galimas srauto greitis čia yra tapatus greičio vidurkiui. Statistikoje nagrinėjant normalinį skirstinį apie šį dydį pasiskirsto galimos kintamųjų reikšmės, o gautas greičio vidurkis įgyja didžiausią reikšmę. Be to, tyrimais pagrįsta greičio ir koncentracijos priklausomybė rodo, kad šią priklausomybę galima aprašyti lognormaliniu skirstiniu [3]. Dėl šios priežasties patogu greičio vidurkį V naudoti nustatant laiko žingsnį Δt. Kitas svarbus žingsnis modeliuojant srautus yra sudaryti tinkamą greičio ir koncentracijos priklausomybę. Pastebima, jog tais atvejais, kai nagrinėjamas kelio atkarpos taškas yra toli nuo nagrinėjamo kelio ruožo pabaigos, greičio priklausomybė nuo koncentracijos yra artima bandymais nustatytajai [4], tačiau nėra tinkama, nes tam tikru laiko momentu didėjant koncentracijai pradeda didėti ir transporto srauto greitis. Tai prieštarauja bandymais nustatytai priklausomybei. Modelis gaunamas nepakankamai tikslus. Tokio netikslumo priežastis yra nevertinamas arba netiksliai vertinamas greičio pokytis g. Šį sprendinio netikslumą galima pašalinti į lygtis įvedant greičio pasiskirstymo funkciją f [4]. Parenku eksponentinę greičio pasiskirstymo funkciją:

$$f_0 = \left(\frac{k_{rib}\eta}{V}\right)\left[\exp\left(-\frac{v}{V}\right)\right]; \tag{10}$$

$$f = \frac{f_0}{\lambda + \gamma v};\tag{11}$$

čia k_{rib} – ribinė koncentracija, kai automobiliai maksimaliai užpildo kelio ruožą ir sudaro spūstis. $\tau = \Delta t$.

$$\lambda = 1 - \gamma V;\tag{12}$$

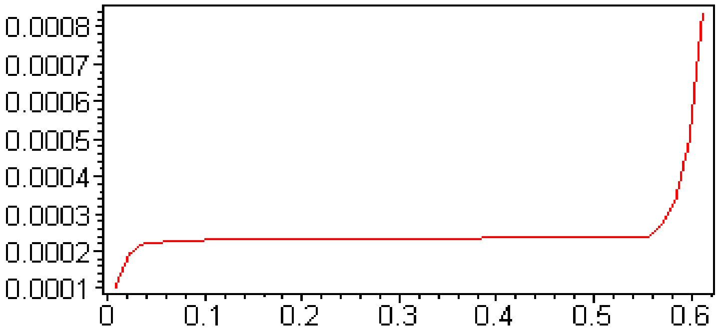
$$\eta = \frac{k}{k_{rib}};\tag{13}$$

$$\gamma = \frac{k_{ribp}\tau\eta^3}{(1-\eta)}.\tag{14}$$

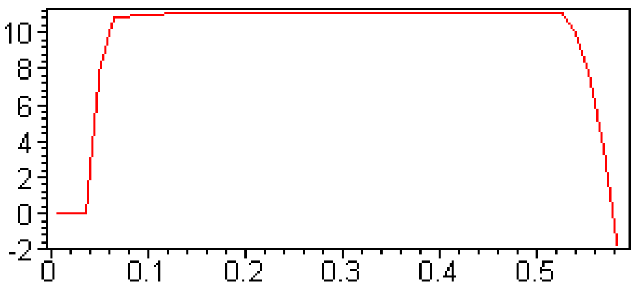
Dabar greičio pokytį g galima išreikšti išraiška:

$$g = \frac{f - f_0}{\Delta t}.\tag{15}$$

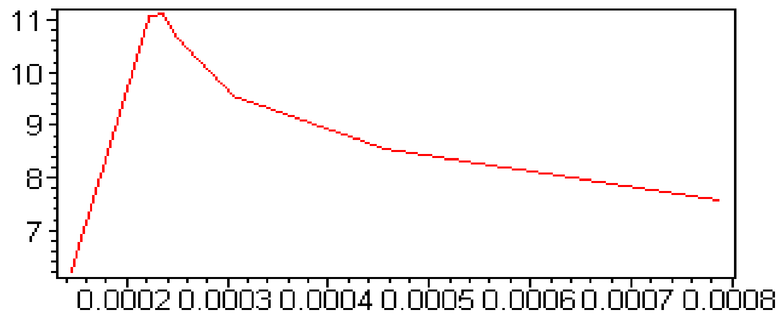
Atlikus matematinį eksperimentą gaunami tokie rezultatai (1, 2, 3, 4, 5 pav.):



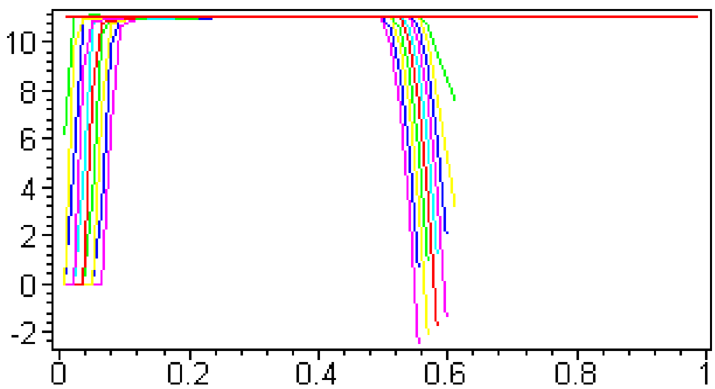
1 pav. Koncentracijos priklausomybė nuo laiko taške *i*



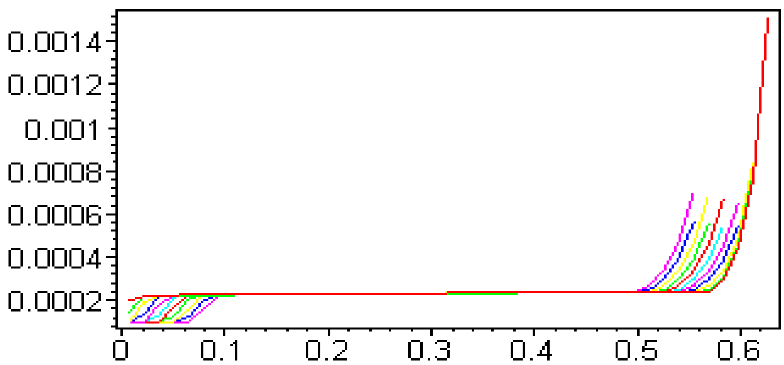
2 pav. Greičio priklausomybė nuo laiko taške *i*



3 pav. Greičio priklausomybė nuo koncentracijos taške *i*



4 pav. Greičio priklausomybė nuo laiko visoje kelio atkarpoje



5 pav. Koncentracijos priklausomybė nuo laiko visame kelio intervale

3. Išvados

Modeliavimo rezultatas atlikus pakeitimus atitinka bandymais patvirtintus rezultatus [4], tačiau nėra visiškai tikslus greičiui artėjant prie nulio, o koncentracijai artėjant prie ribinės koncentracijos. Pageidautinas rezultatas pasiektas atlikus tokius pakeitimus: suvienodinus laiko žingsnio pokytį, įvedus eksponentinę greičio pasiskirstymo funkciją į sprendinį ir tiksliau aprašius greičio pokytį.

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THE STUDY OF THE USAGE OF TRANSPORT TERMINOLOGY ON THE INTERNET

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This article deals with the study of the usage of transport terminology on the Internet using the popular search engine *www.google.lt*. It analyses the frequency of correct and incorrect usage of lexis and word-building. The research showed that mostly there were term forming mistakes. The usage of new loanwords can be justified because most of them are not considered as serious language mistakes. Although, other mistakes can only be explained as the negligence of the authors of the Internet texts.

KELIŲ TRANSPORTO SRAUTŲ MODELIAVIMO GALIMYBĖS

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1. Įvadas

Šiuolaikiniams transporto srautams keliami srauto suvienodinimo ir kelių eismo saugumo užtikrinimo visiems transporto dalyviams reikalavimai, tačiau didėjant transporto priemonių keliuose skaičiui padaugėjo nelaimingų atsitikimų, o aplinka tapo labiau užteršta. Požiūris į kelių transporto sistemą ėmė keistis ir tapo labiau orientuotas į eismo saugumo ir aplinkos užterštumo mažinimo problemas. Kadangi iš pagrindų pakeisti transporto sistemos pasaulyje kol kas neįmanoma ne tik dėl galimybių, bet ir dėl idėjų, naujam transporto tipui, trūkumo, todėl vienintelis tiek ekonomiškai, tiek ekologiškai priimtinas būdas yra esamos transporto sistemos efektyvesnis panaudojimas. Bendru atveju tokią sistemą galima vadinti kelių transporto srautų valdymu.

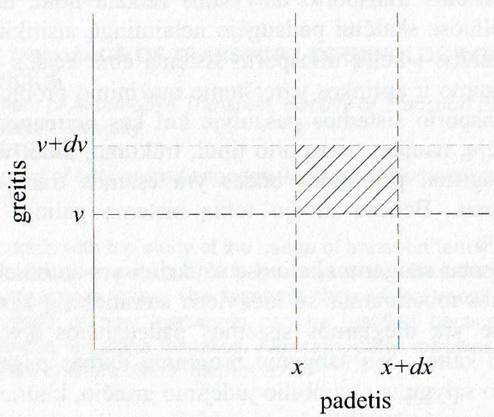
Kitas svarbus momentas saugumui keliuose užtikrinti yra automobilio aktyvaus saugumo sistemos kuriuos tobulinamos su kiekviena automobilių karta. Šiuo metu naujuose automobiliuose yra diegiamos sistemos padedančios išvengti avarinės situacijos dėl vairuotojo kaltės. Šių saugumo programų darbas pagrįstas aplinkos (kelio dangos būklės, oro sąlygų, automobilio judėjimo greičio, kliūties automobilio judėjimo kryptimi buvimo ir kt.) stebėjimu ir vertinimu priimant eismo dalyviams saugiausią sprendimą. Tokių saugumo programų diegimas automobiliuose reikalauja eismo srautų sistemos niuansų išmanymo, vairuotojo elgsenos kelyje prognozavimo, todėl labai svarbu išlieka tobulinti jau sukurtus ir kurti naujus eismo srautų modelius, ypatingą dėmesį skirti vairuotojo elgesio modeliavimui.

Kodėl reikia modeliuoti transporto srautų sistemas? Atsakymu galėtų būti: 1) nuolat kintantis kelių ir gatvių tinklo apkrovimas kuomet pasikliauti projektiniais apkrovimo skaičiavimais nebegalima, todėl kelių ar gatvių tinklo pralaidumo reguliavimui būtina pastoviai perskirstyti tam tikrų gatvių apkrovimą; 2) vairuotojo elgesio prognozavimas – maršruto pasirinkimas, vairavimo tipas, aplinkos vertinimas ir kt.; 3) atsitiktinių faktorių ir fluktacijų, susietų su sezonais, darbo ir išeiginėmis dienomis ir kt., poveikis.

2. Kinetinis modelis

Viena kertinių transporto srauto sąvokų modeliuojant transporto srautus yra [2]: transporto srauto greičio pasiskirstymo funkcija duotuoju laiko momentu duotajame kelio atkarpos taške – $f(x,v,t)$. Šios funkcijos prasmė – tam tikras automobilių skaičius dN duotuoju laiko momentu yra kelio atkarpos intervale tarp x ir $x+dx$ ir greičio intervale tarp v ir dv (1 pav.):

$$dN = f(x,v,t)dxdv; \quad (1)$$



1 pav. Schematinis kelio koordinatės ir greičio ryšys

Čia 1 pav. grafiškai parodyta greičio ir kelio koordinatės pasiskirstymo prasmė, tačiau šis plotelis turi būti pakankamai didelis, t.y. talpinti savyje tokį informacijos (automobilių) kiekį, kad greičio pasiskirstymo funkcija f būtų tolydi visame savo ilgyje, todėl pasirenkama kelio atkarpa turi būti pakankamai ilga.

Yra keletas tipinių greičio pasiskirstymo funkcijų: normalinė, eksponentinė, modifikuota eksponentinė ir kt. Viena jų – eksponentinė užrašoma sekančia išraiška [2]:

$$f = \frac{\left(\frac{c_p\eta}{v_0}\right)\left[\exp\left(-\frac{V}{v_0}\right)\right]}{\lambda+\gamma V}; \quad (2)$$

čia: V – automobilio srauto judėjimo greitis, v_0 – trokštamas automobilio judėjimo greitis.

$$\lambda = 1-\gamma\bar{w} \quad \text{– jautrumo koeficientas;} \quad (3)$$

$$\gamma = \frac{c_p\tau\eta^3}{(1-\eta)} \quad \text{– koeficientas įvertinantis aplenkimo tikimybę.} \quad (4)$$

Formulėse (2, 3) esantys dydžiai: τ – laikas, $\eta = \frac{c}{c_p}$; čia c_p – koncentracija kai lenkimas neįmanomas. Kuomet yra žinoma greičio pasiskirstymo funkcija galima nustatyti kitus svarbius transporto srauto parametrus [2]: pirmas jų yra lokalinė transporto srauto koncentracija ***c(x,t)***:

$$c(x,t) = \int_0^\infty f(x,v,t)dv; \quad (5)$$

antras – globalinė transporto srauto koncentracija ρ:

$$\rho = \int_s c(x,t,s)ds;$$

čia: s –kelio atkarpoje esančių automobilių būsena.

Trečias – transporto srautas $\boldsymbol{q}$ duotajame kelio taške:

$$q = c\bar{v}(x,t) = \int_0^\infty v \cdot f(x,v,t)dv; \quad (6)$$

Modeliuojant transporto srautus ir norint įvertinti skirtingą transporto srautų judėjimo greitį naudojamas greičio dispersijos sąvoka [1]:

$$\Theta = \int_s (V-v)^2 \frac{\rho(t,x,s)}{c(t,x)}ds. \quad (7)$$

Transporto judėjimo lygtis užrašome pasinaudoję Dujų kinetine teorija, arba trumpiau vadinama tiesiog kinetine teorija. Šiuo būdu transporto srautų judėjimas užrašomas dviem pagrindinėmis lygtimis [1]:

srauto vientisumo lygtimi

$$\frac{\partial c}{\partial t}+\frac{\partial cV}{\partial x}=\frac{dc}{dt}; \qquad (8)$$

ir greičio kitimo lygtimi

$$\frac{\partial V}{\partial t}+\frac{\partial V}{\partial x}V+\frac{1}{c}\frac{\partial c\Theta^e}{\partial x}=\left(\frac{dv}{dt}\right)_v+\frac{1}{c}\left(\int_v\left(\frac{d\rho}{dt}\right)dv-V\left(\frac{dc}{ct}\right)\right); \qquad (9)$$

Devintosios lygties dešinėje pusėje esantis pirmasis narys aprašo skirtingais greičiais judančių automobilių judėjimą.

$$\frac{dv}{dt}=\frac{w-v}{\tau_w}; \qquad (11)$$

čia: w – greitis kuriuo norima važiuoti, τ_w – greičių susivienodinimo laikas.

Priėmus, kad laikas τ_w visiems automobiliams vienodas užrašoma išraiška:

$$\left(\frac{dv}{dt}\right)_v=\int_v\frac{\rho}{c}\frac{w-v}{\tau_w}dv=\frac{W-V}{\tau_w}; \qquad (12)$$

Šioje lygtyje narys W (norimas srauto judėjimo greitis) randamas:

$$W=\frac{1}{c}\int_v\omega\rho dv. \qquad (13)$$

Toliau spendžiant transporto srautų uždavinius lygtis 8 ir 9 užrašomos sekančia forma:

$$\frac{\partial}{\partial t}(U)+\frac{\partial}{\partial x}(F(U))=G(U); \qquad (14)$$

Šioje lygtyje esantys vektoriai skleidžiami sekančia forma [1]:

$$U=\begin{pmatrix}u_1\\u_2\end{pmatrix}=\begin{pmatrix}c\\Vc\end{pmatrix}; \qquad (15)$$

$$F(U)=\begin{pmatrix}u_2\\ \frac{u_2^2}{u_1}+u_1\Theta^e(u_1,u_2)\end{pmatrix}; \qquad (16)$$

$$G(U)=\begin{pmatrix}\left(\frac{du_1}{dt}\right)_{elem}\\ u_1\left(\frac{dv}{dt}\right)_v+\int_v v\left(\frac{d\rho}{dt}\right)_{elem}dv\end{pmatrix}; \qquad (17)$$

Lygtis 14 užrašoma sekančia forma:

$$\frac{\partial}{\partial t}(U)+\frac{\partial F}{\partial U}\frac{\partial U}{\partial x}=G(U); \qquad (18)$$

Lygtyje narys $\frac{\partial F}{\partial U}$ – Jakobio matrica, kuri užrašoma sekančiu pavidalu [1]:

$$J(U)=\frac{\partial F}{\partial U}=\begin{bmatrix}0 & 1\\ -\frac{u_2^2}{u_1^2}+\Theta^e+u_1\frac{\partial\Theta^e}{\partial u_1} & 2\frac{u_2}{u_1}+u_1\frac{\partial\Theta^e}{\partial u_2}\end{bmatrix}; \qquad (19)$$

Sudarant transporto srautų lygtis duotoji kelio atkarpa sudalinama į intervalus Δx, kurių centro koordinatės yra

$$x_j=\frac{\Delta x}{2}+j\Delta x;\ \text{kai}\ j=1...(n-1),\ \Delta x=\frac{L_{kelio}}{n};$$

Kiekvieno sudalinto kelio segmento ribinės koordinatės yra:

$$x_{j-\frac{1}{2}}=x_j-\frac{\Delta x}{2};\quad x_{j+\frac{1}{2}}=x_j+\frac{\Delta x}{2};$$

Jeigu sprendžiant transporto srautų uždavinį vertinamos tik segmentą U_j veikiančios kraštinės sąlygos $F(U_{j-1/2})$ ir $F(U_{j+1/2})$ jo ribiniuose taškuose informacijos pokytis segmente U_j po laiko momento Δt užrašomas išraiška:

$$U'_j = U_j - \Delta t \frac{F(U_{j+\frac{1}{2}}) - F(U_{j-\frac{1}{2}})}{\Delta x}; \quad (20)$$

Jeigu sprendžiant transporto srautų uždavinį vertinamos ir segmentą U_j veikiančios išorinės bei vidinės jėgos, aprašomos vektoriumi $G(U_j)$, sistemos sprendinys užrašomas tokia forma:

$$U''_j = U'_j + \Delta t G(U_j); \quad (21)$$

Lygtis 20 ir 21 galima spręsti charakteristikų metodu. Sprendinio tikslumas priklauso nuo koeficiento c, kuris turi tenkinti sąlygą:

$$c < \left(-\tau_w \frac{dV^e}{dc}\right)^{-\frac{1}{2}}; \quad (22)$$

Šis sprendinys gali būti naudojamas modeliuoti transporto srautus tiesiame kelio ruože, o atitinkamai pakoregavus srautus aprašančias lygtis – galima modeliuoti srautus įvertinant ir vairuotojo elgseną.

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POSSIBILITIES OF ROAD TRAFFIC FLOW MODELING

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There in this paper is listed summary of the traffic flow numerical solution based on kinetic traffic flow theory. The behavior of the different traffic flow system participants is included so this numerical solution could be adapted for various situations on straight road, also including the driver behavior.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ЭКСПЛУАТАЦИОННОЙ НАГРУЗКИ НА ЖЕЛЕЗНОДОРОЖНЫЙ ПОЛИГОН В РАЙОНЕ МЕСТНОЙ РАБОТЫ

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Качество математического моделирования эксплуатационной работы на железнодорожном транспорте во многом обуславливается адекватным представлением исходных данных. При этом важно использовать не только достоверные числовые параметры, но и формализованные математические зависимости этих параметров, позволяющие производить всесторонний анализ результатов моделирования и отслеживать поэтапное их изменение.

Управление перевозкой грузов в местном железнодорожном сообщении производится посредством управления вагонопотоками. Грузопотоки $X(t)$ и вагонопотоки $U(t)$ жестко связаны между собой

$$X(t) = o[U(t)], \quad (1)$$

где: $o[U(t)]$ – линейная функция $U(t)$.

Вагонопотоки на железнодорожном полигоне врайоне местной работы можно задать в виде множества

$$U_{pmp} = \bigcup_{i=1}^{n} u_i \{t_i, p_{ijk}, x_i, u_i^{nc}, c_i^{mek}\}, \quad (2)$$